H. UEDA
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UNICITY THEOREMS FOR MEROMORPHIC OR ENTIRE FUNCTIONS

BY HIDEHARU UEDA

1. Let f and g be meromorphic functions. We denote the order of f by ρ_f. In what follows we use the notation $f=a \rightarrow g=a$ in the following sense: Z_n is a zero of $g-a$ of order at least $\nu(n)$ whenever Z_n is a zero of $f-a$ of order $\nu(n)$. If k is a positive integer or ∞, let $E(a, k, f)=\{z \in \boldsymbol{C}: z$ is a zero of $f-a$ of order $\leqq k.\}$, where $\boldsymbol{C}$ is the complex plane. If a belongs to $\bar{\boldsymbol{C}}=\boldsymbol{C} \cup\{\infty\}$, we denote by $\bar{n}_k(r, a, f)$ the number of distinct zeros of order $\leqq k$ of $f-a$ in $|z| \leqq r$ (each zero of order $\leqq k$ is counted only once irrespective of its multiplicity). And we set

$$\bar{N}_k(r, a, f)=\int_0^r \frac{\bar{n}_k(t, a, f)-\bar{n}_k(0, a, f)}{t} dt+\bar{n}_k(0, a, f) \log r .$$

Further we denote by $n_0^{(k)}(r, a ; f, g)$ the number of common zeros of order $\leqq k$ of $f-a$ and $g-a$ in $|z| \leqq r$, and we set

$$N_0^{(k)}(r, a ; f, g)=\int_0^r \frac{n_0^{(k)}(t, a ; f, g)-n_0^{(k)}(0, a ; f, g)}{t} dt+n_0^{(k)}(0, a ; f, g) \log r .$$

In this paper we shall prove some unicity theorems for meromorphic or entire functions.

2. Gopalakrishna and Bhoosnurmath have proved the following theorem in [1].

THEOREM A. *Let f and g be transcendental meromorphic functions. Assume that there exist distinct elements $a_1, \cdots, a_m$ in $\bar{\boldsymbol{C}}$ such that $E(a_i, k_i, f)=E(a_i, k_i, g)$ for $i=1, \cdots, m$; where each k_i is a positive integer or ∞ with $k_1 \geqq \cdots \geqq k_m$, and $\{k_i\}_1^m$ satisfies*

$$\sum_{i=2}^m \frac{k_i}{k_i+1}-\frac{k_1}{k_1+1}>2 .$$

Then $f \equiv g$.

From Theorem A several consequences including a theorem of Nevanlinna [4] are deduced.

THEOREM A_1. *Let f and g be transcendental meromorphic functions. If there*

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