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KODAI MATH. J.
3 (1980), 407—414

QUASIHARMONIC L^p FUNCTIONS ON THE POINCARÉ N-BALL

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By definition, a function q on a Riemannian manifold R is quasiharmonic if $\Delta q=1$, with $\Delta=d\delta+\delta d$ the Laplace-Beltrami operator. There is fairly extensive literature on the existence of quasiharmonic functions in the classes P, B, D or C of functions which are positive, bounded, Dirichlet finite, or bounded Dirichlet finite, respectively. In contrast, very little is known about the existence of quasiharmonic functions in L^p.

Let Q be the class of quasiharmonic functions and set $QX=Q\cap X$ for $X=P$, B, D, C, L^p. For any function class F, denote by O_F the class of Riemannian manifolds which do not carry any nonconstant functions in F, and $\tilde{O}_F$ its complement. The purpose of the present study is to give a criterion for $R\in O_{QL^p}$ and to relate the class O_{QL^p} to some harmonic and quasiharmonic null classes. We also discuss interrelations between the classes O_{QL^p} for various p. For explicit results, we consider these problems on the Poincaré N-ball B_α^N, that is, the unit ball of N-space, $N\geqq 2$, endowed with the metric $ds_\alpha=(1-r^2)^\alpha ds_0$, $r=|x|$, with $\alpha\in\boldsymbol{R}$ and ds_0 the Euclidean metric.

We start by stating, with or without proofs, some auxiliary results, mostly known, to be called Propositions. The new results will be given in Lemmas 1-6 and Theorems 1-3.

§1. Preliminaries

Propositions 1 and 2 on the general behavior of quasiharmonic functions will greatly simplify earlier proofs on characterizing quasiharmonic null classes. Let $(r, \theta)=(r, \theta^1, \cdots, \theta^{N-1})$ be the polar coordinates in $\boldsymbol{R}^N$.

PROPOSITION 1. *Every quasiharmonic function $q(r, \theta)$ on B_α^N can be represented as*

$$q(r, \theta)=q(r)+h(r, \theta)=q_0(r)+c+h(r, \theta),$$

where $h(r, \theta)$ is a harmonic function, c a constant, and

$$q_0(r)=-\int_0^r(1-t^2)^{-(N-2)\alpha}t^{-(N-1)}\int_0^t(1-s^2)^{N\alpha}s^{N-1}ds\,dt.$$

Received August 6, 1979