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NOTES ON LINKS OF COMPLEX ISOLATED SINGULAR POINTS

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§ 1. Introduction.

Let $V \subset C^{n+1}$ be an algebraic variety of codimension d with isolated singular point at the origin, and K be the intersection with a small sphere S^{2n+1} centered at the origin. Then the cobordism class of K in S^{2n+1} is the obstruction to topological smoothing of the singularity and is represented by the element of $\pi_{2n+1}(MU(d))$.

D. Sullivan [3] gave an interesting conjecture that the link K cannot represent an element involving higher order whitehead product in $\pi_{2n+1}(MU(d)) \otimes Q$.

In this note we consider the case when V is a cone over smooth projective variety. Then using M. Larsen's result [2], we shall give an affirmative answer to Sullivan's conjecture. Moreover we can describe the represented class in $\pi_{2n+1}(MU(d)) \otimes Q$ in terms of the chern class of the normal bundle of V in CP^n. This shows that the classical Thom's condition is the only rational obstruction to topological smoothing of the singularity.

§ 2. The rational homotopy type of $MU(d)$.

In this section we shall construct the minimal model of universal Thom space $MU(d)$ according to Sullivan's idea.

Let c_i be the i-th chern class of the universal complex vector bundle, then $H^*(BU(d), Q) \cong Q[c_1 \cdots c_d]$. If $I=(i_1 \cdots i_{d-1})$ is a multi-index, we denote by c_I the monomial $c_1^{i_1} \cdots c_{d-1}^{i_{d-1}}$. Let S be the polynomial ring generated by c_d and $c_d c_I$, and T be the ideal of S generated by forms $(c_d c_I)(c_d c_J)-(c_d)(c_d c_{I+J})$. Then from the cofibration

$$BU(d-1) \to BU(d) \to MU(d)$$

we have the ring isomorphism:

$$H^*(MU(d)) \cong S/T$$

Since the rational homotopy type of $MU(d)$ is the formal consequence of its cohomology ring ([3]), we can construct the minimal model of $MU(d)$ from

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