INFINITESIMAL VARIATIONS OF INVARIANT SUBMANIFOLDS OF A SASAKIAN MANIFOLD

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§ 0. Introduction.

An infinitesimal variation of an invariant (complex) submanifold of a Kaehlerian manifold which carries it into an invariant submanifold is said to be *complex*. An infinitesimal variation is said to be *holomorphic* when it is complex and preserves the complex structure on the invariant submanifold. ([1], [4], [5]). Okumura and two of the present authors [5] proved that an infinitesimal complex conformal variation of a compact orientable submanifold of a Kaehlerian manifold is necessarily isometric and holomorphic and derived a necessary and sufficient condition for a complex variation to be volume-preserving and holomorphic by using an integral formula.

The main purpose of the present paper is to study infinitesimal variations of invariant submanifolds of a Sasakian manifold and to prove theorems analogous to those proved in [4] and [5].

In preliminary §1 we state some properties of invariant submanifolds of a Sasakian manifold.

In §2, we derive fundamental formulas in the theory of infinitesimal variations and study invariant variations, that is, infinitesimal variations which carry an invariant submanifold into an invariant submanifold. In §3 we study f-preserving variations, that is, invariant variations which preserve the tensor field $f_b{}^a$ of the Sasakian structure $(f_b{}^a, g_{cb}, f_b)$ induced on an invariant submanifold.

In §4 we study invariant conformal variations and prove that an invariant conformal fiber-preserving variation of a compact orientable invariant submanifold of a Sasakian manifold is necessarily isometric and hence f-preserving. In the last §5 we prove an integral formula concerning invariant variation and show some of its applications.

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