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# ON THE ORDER OF A ZERO OF THE THETA FUNCTION

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**1. Introduction.** In this paper we shall examine the bounds of the order of a zero of the theta function attached to a compact Riemann surface with a non-trivial conformal automorphism.

At first, we shall give an estimate at the vector of Riemann constants, whose base point is a fixed point of an automorphism. Recently, Farkas [2] has given an estimate from below and some equality conditions when the genus is congruent to one modulo the order of the automorphism. In this paper we shall consider both lower and upper bounds.

Secondly, we shall give an estimate at half periods. Accola [1] has examined them when the surface has an automorphism of order 2 and Farkas [2] has also given some estimates. We shall examine them when an automorphism satisfies a certain condition on its fixed points. Our estimates contain Accola's one and our proof is a modified one of Farkas'. Furthermore, if the genus of the orbit surface of the cyclic group generated by an automorphism is one, we shall give another estimate.

Thirdly, we shall give examples of Riemann surfaces which attain the bounds of the estimates at the vector of Riemann constants.

Lastly, we shall state closing remarks.

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**2. Preliminary.** In this section we state notations and known results. Let $S$ be a compact Riemann surface of genus $g$ ($\geqq 2$) with a canonical homology basis $\alpha_1, \cdots, \alpha_g, \beta_1, \cdots, \beta_g$. Let $\phi_1, \cdots, \phi_g$ be the basis for the space of abelian differentials of the first kind on $S$, which are normalized so that $\int_{\alpha_i} \phi_j = \delta_{ij}\pi i$ where $\delta_{ij}$ is the Kronecker $\delta$. Let $\Omega = (\omega_{ij})$ denote the matrix where $\omega_{ij} = \int_{\beta_i} \phi_j$. It is known that $\Omega$ is symmetric with negative definite real part. Then the first order theta function is defined by

$$\theta(z;\ \Omega) = \sum_m \exp(2{}^t m z + {}^t m \Omega m)$$

where the variable $z$ is $g\times 1$ vector and $m$ runs over all $g\times 1$ vectors with

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