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KŌDAI MATH. SEM. REP.
28 (1977), 284–299

# ON THE CHARACTERISTIC FUNCTIONS OF QUATERNION KÄHLERIAN SPACES OF CONSTANT Q-SECTIONAL CURVATURE

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## 1. Introduction.

In Riemannian geometry, one of interest problems is the determination of of all Einstein spaces. But it is difficult to solve this problem, for we cannot even classify the class of harmonic spaces defined by analytic conditions, which is less general than that of Einstein spaces. The problems of classifying harmonic spaces, those of finding canonical forms for their metrics and those of determining its characteristic functions have taken up by E. T. Copson and H. S. Ruse [1], A. G. Walker [9], [10], [11], A. Lichnérowicz [5], T. J. Willmore [13], A. J. Ledger [4], S. Tachibana [7], the author [11] and others.

Its typical examples are the following: (1) Euclidean space $R^n$, (2) sphere $S^n$, (3) real projective space $RP^n$, (4) complex projective space $CP^m$, (5) quaternion projective space $HP^m$ and (6) the Cayley projective plane $\mathfrak{C}P(2)$ ([4], [6]). The characteristic functions of $S^n$, $RP^n$ and $CP^m$ have been already obtained as follows ([6], [7]): An $n$-dimensional space of constant curvature ($k\neq 0$) is characterized as a harmonic Riemannian space with characteristic function

$$f(\Omega)=1+(n-1)\sqrt{2k\Omega}\cot\sqrt{2k\Omega}$$

and a $2m$-dimensional space of constant holomorphic curvature ($k\neq 0$) as a harmonic Kählerian space with characteristic function

$$f(\Omega)=1+(2m-1)(ls)\cot(ls)-(ls)\tan(ls)$$

or

$$f(\Omega)=1+(2m-1)(ls)\coth(ls)+(ls)\tanh(ls)\,,$$

according to $k=4l^2$, or $k=-4l^2$ respectively, where $s$ means the geodesic distance and $\Omega=(1/2)s^2$.

In spite of these facts, the characteristic functions of $HP^m$ and $\mathfrak{C}P(2)$ have

Received Dec. 18, 1975.
*) This work was partially supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, which is gratefully acknowledged.