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ON ADMISSIBLE DATA OF CAUCHY PROBLEM FOR SECOND ORDER EQUATIONS WITH CONSTANT COEFFICIENTS

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§1. Introduction.

Let $P(D)$ be a linear partial differential operator with constant coefficients and let X be a hyperplane. It is well known that if $P(D)$ is hyperbolic with respect to N which is a normal vector of X, then the Cauchy problem for $P(D)$ with any given C^∞-data on X has unique solution defined on both sides of X (see. [2] Chap. V). On the contrary, if $P(D)$ is not hyperbolic with respect to N, Cauchy data of the Cauchy problem which have solutions make a proper subset of the class of C^∞-functions. In this note we shall consider the properties of admissible data (see. Definition 1.1) for such not necessary hyperbolic Cauchy problems. We shall restrict our considerations to operators of second order with real constant coefficients.

Let $P(D)=P(D_x, D_y)$ be a linear partial differential operator acting on $\boldsymbol{R}^{n+1}$ with the following form:

$$(1.1)\qquad P=P(D_x, D_y)=a_0D_y{}^2+\sum_{i=1}^n a_iD_yD_{x_i}+\sum_{i,j=1}^n a_{ij}D_{x_i}D_{x_j}+b_0D_y+\sum_{i=1}^n b_iD_{x_i}+c\,,$$

where a_0, a_i, a_{ij}, b_0, b_i, c are real constant coefficients, $x=(x_1, \cdots, x_n)\in\boldsymbol{R}^n$, and $D_y=\partial/\partial y$, $D_{x_i}=\partial/\partial x_i$ $(i=1, \cdots, n)$. Without loss of generality we can assume that $a_{ij}=a_{ji}$. We note the characteristic polynomial of $P(D)$ as follows:

$$(1.2)\qquad P(\xi, \tau)=a_0\tau^2+\sum_{i=1}^n a_i\tau\xi_i+\sum_{i,j=1}^n a_{ij}\xi_i\xi_j+b_0\tau+\sum_{i=1}^n b_i\xi_i+c\,.$$

In what follows, we always assume that the hyperplane

$$X=\{(x, y)\in\boldsymbol{R}^{n+1}\,;\,y=0\}$$

is non-characteristic for $P(D)$ (i.e. $P(0, 1)=a_0\neq 0$). Consider the following Cauchy problem: For given functions $f(x)$, $g(x)$ which are defined in some neighbourhood of 0 in X, find the solution $u=u(x, y)$ of

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