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ON SEMI-SYMMETRIC METRIC φ-CONNECTIONS IN A SASAKIAN MANIFOLD

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§ 0. Introduction.

Let M be an n-dimensional Riemannian manifold covered by a system of coordinate neighborhoods $\{U; x^h\}$ and with the fundamental metric tensor g_{ji}, where and in the sequel the indices $h, i, j, \cdots$ run over the range $\{1, 2, \cdots, n\}$. A linear connection D with components Γ_{ji}^h of M is said to be semi-symmetric if its torsion tensor $S_{ji}{}^h = \Gamma_{ji}^h - \Gamma_{ij}^h$ is of the form $S_{ji}{}^h = \delta_j^h p_i - \delta_i^h p_j$, p_i being a 1-form and is said to be metric if it satisfies $D_k g_{ji} = 0$.

The components of a semi-symmetric metric connection in a Riemannian manifold are given by [3]

$$\Gamma_{ji}^h = \left\{ {h \atop j\ i} \right\} + \delta_j^h p_i - g_{ji} p^h, \tag{0.1}$$

$\left\{ {h \atop j\ i} \right\}$ being the Christoffel symbols formed with g_{ji} and $p^h = p_t g^{th}$. One of present authors [3] proved that: In order that a Riemannian manifold admits a semi-symmetric metric connection whose curvature tensor vanishes, it is necessary and sufficient that the Riemannian manifold is conformally flat.

Let M be a Kaehlerian manifold with Hermitian metric g_{ji} and almost complex structure tensor $F_i{}^h$. A linear connection D with components Γ_{ji}^h of M is called a complex conformal connection if it satisfies

$$D_k e^{2p} g_{ji} = 0, \qquad D_k e^{2p} F_{ji} = 0, \qquad (F_{ji} = F_j{}^t g_{ti})$$

and

$$\Gamma_{ji}^h - \Gamma_{ij}^h = -2F_{ji} q^h$$

for a certain scalar p and a vector field q^h.

The components of a complex conformal connection are given by [4]

$$\Gamma_{ji}^h = \left\{ {h \atop j\ i} \right\} + \delta_j^h p_i + \delta_i^h p_j - g_{ji} p^h + F_j{}^h q_i + F_i{}^h q_j - F_{ji} q^h, \tag{0.2}$$

where $p_i = \partial_i p$, $p^h = p_t g^{th}$, $q_i = -p_t F_i{}^t$ and $q^h = q_t g^{th}$, ∂_i denoting the partial derivation with respect to x^i. One of the present authors [4] proved that: If, in an n-dimensional Kaehlerian manifold ($n \geqq 4$), there exists a scalar function p

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