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φ-TRANSFORMATIONS ON A K-CONTACT RIEMANNIAN MANIFOLD

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§ 0. **Introduction.** It is very interesting to make reserches on the subject of manifolds admitting a tensor field invariant under a certain transformation. Now, S. Tanno has studied φ-transformations on almost contact Riemannian manifolds and given several important conclusions ([4]). The main purpose of the present paper is to prove Theorems 2.2, 3.1, 4.2 and 4.4.

§ 1. **Preriminaries.** Let M be a $(2n+1)$-dimensional differentiable manifold satisfying the second axiom of countability. In this paper, manifolds, geometric objects and mappings we consider are assumed to be differentiable and of class C^∞. If there exists a tensor field $\varphi_j{}^i$ of type (1.1), contravariant and covariant vector fields ξ^i and η_i on M which satisfy the following conditions:

$$\xi^i\eta_i=1, \tag{1.1}$$

$$\varphi_r{}^i\varphi_j{}^r=-\delta_j{}^i+\xi^i\eta_j, \tag{1.2}$$

then M is said to have an almost contact structure and called an almost contact manifold. The suffices $k, j, \cdots, i$ run over the range $\{1, 2, \cdots, 2n+1\}$ and the summation convension will be used. For an almost contact structure the following identities are established ([3]):

$$\varphi_r{}^i\xi^r=0, \qquad \eta_r\varphi_j{}^r=0. \tag{1.3}$$

Let M be an almost contact manifold. Then there exists a positive definite Riemannian metric g_{ji} such that

$$\eta_i=g_{ir}\xi^r, \tag{1.4}$$

$$g_{sr}\varphi_j{}^s\varphi_i{}^r=g_{ji}-\eta_j\eta_i. \tag{1.5}$$

Such a metric tensor g_{ji} is called an associated Riemannian metric with the given almost contact structure. If a differentiable manifold M admits tensor fields $(\varphi_j{}^i, \xi^i, \eta_i, g_{ji})$ such that g_{ji} is a Riemannian metric associated with the almost contact structure, then M is called an almost contact Riemannian manifold.

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