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A BOUND FOR THE NUMBER OF AUTOMORPHISMS OF A FINITE RIEMANN SURFACE

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1. Introduction.

An automorphism of a Riemann surface X is a 1—1 conformal mapping of X onto itself. Let $N(g)$ be the order of the largest group of automorphisms a compact Riemann surface of genus g can admit. Similarly we denote by $N(g, k)$ the maximal order of an automorphism group of a finite (i. e. compact bordered) Riemann surface of genus g with k boundary components; in particular $N(g, 0) = N(g)$. In this paper the following bound will be proved:

$$N(g, k) \geqq \max(6, (g/2)^{1/2}) \qquad \text{for all} \quad g, k \geqq 0,$$

and 6 cannot be replaced by a larger constant. This improves the lower bound $N(g, k) \geqq 4$ given by R. Tsuji [8].

2. Known results.

A. Hurwitz [3] proved $N(g) \leqq 84(g-1)$ for $g \geqq 2$ and A. M. Macbeath [5] showed that this bound is attained for infinitely many values of g. R. D. M. Accola [1] and C. Maclachlan [6] proved independently that $N(g) \geqq 8(g+1)$ and this lower bound is also exact for an infinite number of g's.

Automorphisms of finite Riemann surfaces were studied by M. Heins, K. Oikawa, R. Tsuji, T. Kato and others. M. Heins (for the case $g=0$) and K. Oikawa proved that $N(g, k)$ equals the maximal order of an automorphism group of a compact Riemann surface X_g of genus g being punctured in k distinct points ([2], [7]). They showed that a finite Riemann surface $X_{g,k}$ of genus g with k boundary components can be imbedded into a compact Riemann surface X_g in such a manner that the automorphisms of $X_{g,k}$ can be continued to automorphisms of the punctured surface $X'_{g,k} = X_g - \{P_1, \cdots, P_k\}$ where the P_j are suitably chosen distinct points of X_g. On the other hand if we endow the Riemann surface X_g $(g>1)$ with the Poincaré metric then the automorphisms of X_g are isometries. Hence there are discs D_j with equal radius and midpoint P_j such that the automorphisms of $X'_{g,k}$ are also automorphisms of the finite Riemann surface $X_{g,k} = X'_{g,k} - \cup D_j$. The cases $g=0, 1$ can be treated similarly.

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