K. KOMATSU
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ON THE ADELE RINGS OF ALGEBRAIC NUMBER FIELDS

BY KEIICHI KOMATSU

Introduction.

Let Q be the rational number field, $\bar{Q}$ the algebraic closure of Q and k ($k \subset \bar{Q}$) an algebraic number field of finite degree. Let $\zeta_k(s)$ be the Dedekind zeta-function of k, k_A the adele ring of k and G_k the Galois group of $\bar{Q}/k$ with Krull topology. We adopt similar notations for an algebraic number field k' ($k' \subset \bar{Q}$) of finite degree. If the extension k/Q is a finite Galois extension and if $\zeta_k(s)=\zeta_{k'}(s)$, then $k=k'$ (cf. Lemma 2). The Lemma 7 of [3] shows that $k_A \cong k'_A$ implies $\zeta_k(s)=\zeta_{k'}(s)$ (cf. Corollary of Lemma 3). We also proved that $G_k \cong G_{k'}$ implies $\zeta_k(s)=\zeta_{k'}(s)$ (cf. [6] or [4]). From the above results, it is natural and interesting to consider whether, for any algebraic number fields k and k' of finite degree, $k_A \cong k'_A$ implies $k \cong k'$ and whether $G_k \cong G_{k'}$ implies $k \cong k'$. In Theorem 1, we shall show that there exist algebraic number fields k and k' of finite degree satisfying the following conditions:

1) $\zeta_k(s)=\zeta_{k'}(s)$.
2) $k_A \not\cong k'_A$.

Furthermore, in Theorem 2, we shall show that there exist algebraic number fields k and k' of finite degree satisfying the following conditions:

1) $k_A \cong k'_A$.
2) $G_k \not\cong G_{k'}$.

This also shows that there exist algebraic number fields k and k' of finite degree satisfying the following conditions:

1) $k_A \cong k'_A$.
2) $k \not\cong k'$.

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Notation and terminology.

Throughout this paper, Q and Z denote the rational number field and the rational integer ring respectively. An algebraic number field always means an algebraic number field of finite degree, an integer means a rational integer and a prime number means a rational prime number. For an algebraic number field

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