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ON THE VALUE DISTRIBUTION OF ENTIRE FUNCTIONS OF ORDER LESS THAN ONE

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§1. Tsuzuki [4] proved the following;

THEOREM A. *Let $f(z)$ be an entire function of order less than one and $\{w_n\}_{n=1}^{\infty}$ be a sequence such that $|w_n|\to\infty$ as $n\to\infty$. Suppose that there exists ω such that $0<\omega<\pi/2$ and all the roots of the equations*

$$f(z)=w_n \qquad (n=1, 2, \cdots)$$

lie in the angle $A(\omega)=\{z;\ |\arg z-\pi|<\omega\}$. Then $f(z)$ is linear.

The purpose of this note is to extend Theorem A and to prove the following.

THEOREM. *Let $f(z)$ be an entire function of order less than one and $\{w_n\}_{n=1}^{\infty}$ be a sequence such that $|w_n|\to\infty$ as $n\to\infty$. Suppose that all the roots of the equations*

$$f(z)=w_n \qquad (n=1, 2, \cdots)$$

lie in the upper half plane $\operatorname{Im} z\geqq 0$. Then $f(z)$ is a polynomial of degree not greater than two.

§2. **Proof of Theorem.** Suppose that $f(z)$ satisfies the conditions of Theorem and that $f(z)$ is transcendental. Without loss of generality, we may suppose that $w_1=0$, $f(0)\neq 0$ and we have

$$f(z)=\lambda\prod_{j=1}^{\infty}\left(1-\frac{z}{z_j}\right)$$

where $\lambda\ (\neq 0)$ is a constant. Choose ω and η such that $0<\omega<\pi/2$, $\eta=\pi/2-\omega$. Then we have

$$f(z)=\lambda f_1(z)f_2(z)$$

where

$$f_1(z)=\prod_{j_1=1}^{\infty}\left(1-\frac{z}{z_{j_2}}\right) \quad (\eta<\arg z_{j_1}<\pi-\eta),$$

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