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CERTAIN HYPERSURFACES IN THE EUCLIDEAN SPHERE

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§ 0. Introduction.

It has been proved by R. Osserman [6] that if the mean curvature vector of a surface S in the Euclidean space E^3 is always orthogonal to a fixed direction, then S is either a minimal surface, or else a locally cylindrical surface with its generator parallel to the fixed direction.

In this paper, we consider a unit sphere S^{n+1} in the Euclidean space E^{n+2}, and study about a hypersurface M^n in S^{n+1} whose mean curvature vector is always orthogonal to a fixed direction.

We first show that when M is complete, M must be a minimal hypersurface (Theorem I). The necessity of completeness will be discussed in § 2.

We next show that in the case $n=2$, M is either a minimal surface, or else a locally cylindrical surface in S^3, by the latter we mean some open piece of such a surface as is generated by a family of semi-great circles through a fixed pair of antipodal points of S^3 (Theorem II). This corresponds exactly to the result of R. Osserman.

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§ 1. A result in the complete case.

In this section, we prove the following:

THEOREM I. *Let M be an n-dimensional complete Riemannian manifold isometrically immersed in S^{n+1}. If the mean curvature vector of M is always orthogonal to a fixed direction, then M is a minimal hypersurface.*

Proof. As minimality is a local property, we may assume M to be orientable. Without loss of generality, consider S^{n+1} as the unit sphere in E^{n+2} with center at the origin, and let $f: M \to S^{n+1}$ be the immersion in the theorem. For $p \in M$, $x_{f(p)}$ denotes the position vector of $f(p) \in S^{n+1}$ in E^{n+2}, and $T_p(M)$ is the tangent space of M at p, usually identified with $f_*(T_p(M))$. We denote by $\langle\, , \rangle$ the metric on E^{n+2}, S^{n+1} and M without distinction, and by D, $\tilde{\nabla}$ and ∇ the

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