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ASYMPTOTIC BEHAVIOR AND DEGENERACY OF BIHARMONIC FUNCTIONS ON RIEMANNIAN MANIFOLDS

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One of the most fascinating results in harmonic classification theory is the is the identity $O_{HD}^{N}=O_{HC}^{N}$, where H stands for the class of harmonic functions h, $\Delta h=0$, with $\Delta=d\delta+\delta d$ the Laplace-Beltrami operator, and HD, HC are the subclasses of functions which are Dirichlet finite, or bounded Dirichlet finite, respectively. For any class F of functions, O_F, $\tilde{O}_F$ denote the classes of Riemannian manifolds on which $F\subset \boldsymbol{R}$ or $F\not\subset \boldsymbol{R}$ respectively, and O_F^N, $\tilde{O}_F^N$ are the corresponding subclasses of manifolds of dimension $N\geqq 2$.

A striking phenomenon in biharmonic classification theory is that, in contrast with the harmonic case, the inclusion $O_{H^2D}\subset O_{H^2C}$ is strict, with H^2 the class of nonharmonic biharmonic functions. This has been, however, known only in the 2-dimensional case, in which it was established by undoubtedly the most intricate counterexample in all classification theory (Nakai-Sario [6]). The technique of complex analysis used therein is not available for an arbitrarily high dimension.

Combining certain recent results in the biharmonic classification of the Poincaré N-ball for the subclasses H^2D, H^2B of H^2 functions which are Dirichlet finite or bounded, respectively (Hada-Sario-Wang [2], [3]), one can draw the conclusion that $O_{H^2D}^N\subset O_{H^2C}^N$ is strict for $N\geqq 5$. However, for $N=3, 4$, the reasoning fails and the question remains unsettled.

The first purpose of the present paper is to give a complete and unified solution to this problem by proving the strict inclusion

$$O_{H^2D}^N<O_{H^2C}^N$$

for any dimension $N\geqq 2$. We shall, in fact, show more generally that $O_{H^2B}^N\not\subset O_{H^2D}^N$. On the other hand, from recent results on the Poincaré N-ball (Hada-Sario-Wang [2], [3]), we infer that $O_{H^2D}^N\not\subset O_{H^2B}^N$. In summary, we have the following string of strict inclusion relations:

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