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ON CONFORMALLY FLAT SPACES WITH DEFINITE RICCI CURVATURE II

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1. Introduction. There is a formal similarity between the theory of hypersurfaces and conformally flat d-dimensional spaces of constant scalar curvature provided $d \geqq 3$. For, then, the symmetric linear transformation field Q defined by the Ricci tensor satisfies the "Codazzi equation"

$$(\nabla_X Q)Y = (\nabla_Y Q)X .$$

This observation together with the technique and results in [2] and [3] yields the following statement.

THEOREM. *Let M be a compact conformally flat manifold with definite Ricci curvature. If the scalar curvature r is constant and* $\operatorname{tr} Q^2 \leqq r^2/d-1$, $d \geqq 3$, *then M is a space of constant curvature.*

The corresponding result for hypersurfaces is due to M. Okumura [3].

COROLLARY. *A 3-dimensional compact conformally flat manifold of constant scalar curvature whose sectional curvatures are either all negative or all positive is a space of constant curvature.*

Note that, in general $\operatorname{tr} Q^2 \geqq r^2/d$ with equality, if and only if, M is an Einstein space.

Examples of compact negatively curved space forms are given in the paper by A. Borel [1].

2. Definitions and formulas. Let (M, g) be a Riemannian manifold with metric tensor g. The curvature transformation $R(X, Y)$, $X, Y \in M_m$— the tangent space at $m \in M$, and g are related by

$$R(X, Y) = \nabla_{[X,Y]} - [\nabla_X, \nabla_Y] ,$$

where ∇_X is the operation of covariant differentiation with respect to X defined in terms of the Levi-Civita connection. In terms of a basis $X_1, \cdots, X_d$ of M_m we set

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