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ON CONFORMAL DIFFEOMORPHISMS OF 4-DIMENSIONAL RIEMANNIAN MANIFOLDS

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Introduction. Let M and M^* be n-dimensional connected Riemannian manifolds with metric tensor fields g and g^* respectively, and consider a conformal diffeomorphism f of M into M^*. Then the metric tensor fields are related by

$$g^*=\frac{1}{\rho^2}g,$$

where ρ is a positive-valued scalar field on M and said to be associated with f.

In his previous paper [3], the present author proved the following theorems, the first of which is of local character and the second of global character:

THEOREM A. *Assume that M and M^* are Riemannian manifolds of dimension $n\geqq 4$, M is the Pythagorean product of two Riemannian manifolds M_1 and M_2 of dimension n_1 and n_2 respectively, and the Ricci tensor of M^* is parallel. If there is a non-homothetic conformal diffeomorphism of M into M^* such that the associated scalar field ρ depends on both M_1 and M_2 in an open subset in M, then both the parts M_1 and M_2 of M are Einstein manifolds, except the case $n_1=n_2=2$, and the scalar curvatures κ_1 and κ_2 of the parts possess one of the following properties:*

1) $\kappa_1=-\kappa_2=k$, *k being a non-zero constant,*
2) $\kappa_1=k$ *and M_2 is one-dimensional, $\kappa_2=0$,*
3) $\kappa_1=\kappa_2=0$ *so that M is an Einstein manifold of zero scalar curvature.*

THEOREM B. *In addition to the assumptions of the theorem above, we assume that M and M^* are complete and M is reducible in place of being the Pythagorean product. Then there exists no non-homothetic conformal diffeomorphism of M onto M^* such that the associated scalar field ρ depends on both M_1 and M_2 in an open subset in M.*

The purpose of this paper is to discuss conformal diffeomorphisms in the exceptional case $n_1=n_2=2$ of Theorem A and to prove the following theorem of local character:

THEOREM. *Assume that M is the Pythagorean product $M_1\times M_2$ of two-dimensional manifolds M_1 and M_2 with metric tensor g_1 and g_2 respectively and*

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