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THE AXIOM OF COHOLOMORPHIC 3-SPHERES IN AN ALMOST TACHIBANA MANIFOLD

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§1. **Introduction.** Let M be an almost Hermitian manifold with metric tensor $\langle,\rangle$, Riemannian connection ∇, and almost complex structure J. A 2-plane φ is called holomorphic (resp. totally real (or called antiholomorphic)) if $J\varphi=\varphi$ (resp. if $J\varphi$ is prependicular to φ), where we mean an r-dimensional linear subspace of tangent space by r-plane. K. Yano and I. Mogi [9] (resp. B. Y. Chen and K. Ogiue [1]) proved that a Kaehlerian manifold with the axiom of holomorphic 2-planes (resp. the axiom of totally real 2-planes) is a complex space form. A 3-plane is called coholomorphic if it contains a holomorphic 2-planes a holomorphic 2-plane φ. It is clear that a coholomorphic 3-plane also contains a totally real 2-plane. Recently, B. Y. Chen and K. Ogiue [2] have considered the axiom of coholomorphic 3-spheres as follows: *For each point of* $x\in M$ *and each coholomorph* 3*-plane* π, *there exists a* 3*-dimensional, totally umbilical submanifold* N *such that* $x\in N$ *and* $T_x(N)=\pi$.

The purpose of this is to study an almost Tachibana manifold satisfying the axiom of coholomorphic 3-spheres and to prove the following:

THEOREM. *Let M be an n-dimensional non-Kaehlerian almost Tachibana manifold satisfying* $\|(\nabla_X J)(Y)\|^2=constant$ *for all orthonormal vectors X and Y that span a totally real* 2*-plane. If M admits the axiom of coholomorphic* 3*-spheres, then M is* 6*-dimensional manifold of constant curvature* $C>0$.

§2. **Submanifold.** Let N be a submanifold of M, and ∇ and ∇' be the covariant differentiations on M and N respectively. Then the second fundamental forms B of the immersion is defined by $B(X, Y)=\nabla_X Y-\nabla'_X Y$, where X and Y are vector fields tangent to N. B is a normal bundle valuded symmetric 2-form on N. For a vector field ξ normal to N we write $\nabla_X\xi=-A_\xi(X)+D_X\xi$, where $-A_\xi(X)$ (resp. $D_X\xi$) denotes the tangential (resp. normal) component of $\nabla_X\xi$. The submanifold N is said to be totally umbilical if $B(X, Y)=\langle X, Y\rangle H$, where H is the mean curvature vector of N.

For the second fundamental form B of N in M we define the covariant derivative, denoted by $\bar{\nabla}_X B$, to be

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