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ON HARMONIC DIFFERENCE FORMS ON A MANIFOLD

Dedicated to Professor Yûsaku Komatu on his 60th birthday

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Introduction.

In the present note we aim to obtain an orthogonal decomposition theorem of difference forms on a *polyangulation* of a 3-dimensional manifold which is analogous to de Rham-Kodaira's theory on a Riemannian manifold.

In the previous paper [6], we concerned ourselves with the problem of constructing a theory of discrete harmonic and analytic differences on a polyhedron and the problem of approximating harmonic and analytic differentials on a Riemann surface by harmonic and analytic differences respectively, where our definition of a polyhedron differs from the ordinary one based on a triangulation and admits also a polygon and a lune as 2-simplices (cf. § 1. **1** of [6]). In order to set the definitions of a conjugate difference, we introduced concepts of a conjugate polyhedron and a complex polyhedron. In the present note, we shall also introduce similar concepts of a conjugate polyhedron and a complex polyhedron (cf. § 1. **3**) on a 3-dimensional manifold, and we shall show that on such a complex polyhedron a theory of harmonic difference forms analogous to de Rham-Kodaira's theory on Riemannian manifold is obtained.

§ 1. Foundation of topology.

1. Polyangulation. Let E^3 be the 3-dimensional euclidean space. By a *euclidean 0-simplex* we mean a point on E^3. By a *euclidean 1-simplex* we mean a closed line segment or a closed circular arc. By a *euclidean 2-simplex* we mean a closed polygon on a hyperplane or a convex surface, surrounded by a finite number ($\geqq 2$) of segments and circular arcs. A lune (biangle) and a triangle are also admitted as a euclidean 2-simplex. By a *euclidean 3-simplex* we mean a closed convex polyhedron surrounded by a finite number ($\geqq 2$) of such polygons (euclidean 2-simplices). A dihedron and a trihedron (closed convex polyhedra surrounded by two polygons and three ones respectively) are also admitted as a euclidean 3-simplex.

Let F be a 3-dimensional orientable manifold. By an *n-simplex* s^n ($n=0, 1, 2, 3$) on F we mean a pair of a euclidean n-simplex e^n and a one-to-one bi-

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