N. SUITA
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ON A METRIC INDUCED BY ANALYTIC CAPACITY II

Dedicated to Professor Yûsaku Komatu on his 60th birthday

BY NOBUYUKI SUITA

1. Introduction. Let Ω be a plane region having nonconstant bounded analytic functions. Let $c_B(\zeta)$ be the least upper bound of $|f'(\zeta)|$, $\zeta\in\Omega$ in the class of bounded analytic functions satisfying $|f|\leqq 1$ in Ω. In our earlier paper [5] of these reports we have proved that the curvature in the metric $ds_B=|c_B(\zeta)||d\zeta|$ is not greater than -4, by making use of a supporting metric due to Ahlfors [1].

In the present paper we will show that a method of Bergman [3] provides the same result and further a more precise estimation $\kappa(\zeta)<-4$ for a finitely connected region bounded by more than one curves.

2. Extremal problems. Let Ω be a plane region bounded by a finite number of analytic Jordan curves. The class of analytic functions f such that $|f(z)|^2$ has a harmonic majorant in Ω is called the *Hardy class* of index two, denoted by $H_2(\Omega)$. Every function f in $H_2(\Omega)$ has a non-tangential boundary value almost everywhere on the boundary $\partial\Omega$ of Ω which will be denoted by the same notation $f(z)$, $z\in\partial\Omega$. The function $f(z)$ is measurable and square integrable on $\partial\Omega$ [4]. We define the inner product (f, g) of f and $g\in H_2(\Omega)$ by

$$(f, g)=\int_{\partial\Omega} f(z)\overline{g(z)}|dz| .$$

Then $H_2(\Omega)$ becomes a Hilbert space. There exists the Szegö kernel function $k(z, \bar{\zeta})$ in $H_2(\Omega)$ which is characerized by the reproducing property:

$$f(\zeta)=\int_{\partial\Omega} f(z)\overline{k(z, \bar{\zeta})}|dz| , \tag{1}$$

for $f\in H_2(\Omega)$ [3]. The following problems were dealt with by Bergman [3].

Consider two extremal problems:

I) Minimize $\|f\|^2=(f, f)$ in the subclass of $H_2(\Omega)$ each member of which satisfies $f(\zeta)=1$ for $\zeta\in\Omega$.

II) Minimize $\|f\|^2$ in the subclass of $H_2(\Omega)$ each member of which satisfies $f(\zeta)=0$ and $f'(\zeta)=1$.

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