I. SATO
KŌDAI MATH. SEM. REP
26 (1975), 230—239

ON HYPERSURFACES IN EVEN DIMENSIONAL CONTACT RIEMANNIAN MANIFOLDS

BY ISUKE SATO

Introduction. An even dimensional differentiable manifold $\tilde{M}$ is called an even dimensional contact manifold, if it admits a 1-form η such that $(d\eta)^n \neq 0$, where $\dim \tilde{M}=2n$. Then there exists naturally an almost Kählerian structure on $\tilde{M}$. Such a manifold was recently studied from the differential geometric point of view by K. Yano and Y. Mutō [8], [9], T. Nagano [1] and S. Sasaki [3].

In the present paper we study hypersurfaces of an even dimensional contact Riemannian manifold. In § 1 we recall first of all the definition of even dimensional contact Riemannian manifolds and some identities which hold in such manifolds and after some preliminaries, § 2 contains some identities which hold for hypersurfaces in an even dimensional contact Riemannian manifold. In § 3 we prove that if the hypersurface satisfies certain condition, it admits a contact structure. In § 4 an integral formula is obtained for closed hypersurfaces with constant mean curvature and applying the integral formula we prove that, under certain conditions, the hypersurface in question is totally umbilical. Finally in § 5 we consider an even dimensional contact Riemannian manifold in which the structure vector field ξ is contravariant almost analytic and study a hypersurface in this manifold.

1. Even dimensional contact Riemannian manifolds.

A $2n$-dimensional differentiable manifold $\tilde{M}$ is said to have an even dimensional contact structure and called an even dimensional contact manifold if there exists a 1-form η, to be called the contact form, on $\tilde{M}$ such that

$$(d\eta)^n \neq 0 \tag{1.1}$$

everywhere on $\tilde{M}$, where $d\eta$ is the exterior derivative of η.

In terms of local coordinate $\{y^\kappa\}$ of $\tilde{M}$ the contact form η and its exterior derivative are expressed as

$$\begin{aligned} \eta &= \eta_\lambda dy^\lambda , \\ d\eta &= F_{\lambda\mu} dy^\lambda \wedge dy^\mu , \end{aligned} \tag{1.2}$$

Received Sept. 6, 1973.