F. BRICKELL AND K. YANO
KŌDAI MATH. SEM. REP.
26 (1974), 22-28

CONCURRENT VECTOR FIELDS AND MINKOWSKI STRUCTURES

BY FREDERICK BRICKELL AND KENTARO YANO

§1. Concurrent vector fields. We make the general assumption that all the differentiable manifolds and geometric objects which we use are of class C^{∞}. Let M be a differentiable manifold and ∇ a linear connection on M. A vector field Λ on M is *concurrent* with respect to ∇ if

$$\nabla_u \Lambda = u$$

for all vectors u tangent to M. ([4])

Example. Let V be a real vector space of dimension n and choose a basis $E_1, \cdots, E_n$ for V. A vector $v \in V$ can be expressed uniquely as

$$v = \sum_i x^i(v) E_i, \qquad i=1, \cdots, n$$

and the *standard* chart $(x^1, \cdots, x^n)$ defines a manifold structure on V which is independent of the particular basis chosen. The vector field $\sum_i x^i(\partial/\partial x^i)$ also is independent of the chosen basis and we call it the *radial vector field* on V. The conditions

$$\nabla_{\partial/\partial x^i}(\partial/\partial x^j) = 0, \qquad i, j=1, \cdots, n$$

determine a complete linear connection on V which we call the *standard* connection on V. The radial vector field is concurrent with respect to the standard connection.

A riemannian metric g on M determines a unique connection on M called a riemannian connection. We say that Λ is concurrent with respect to g if it is concurrent with respect to the corresponding riemannian connection.

Example. Let $x^1, \cdots, x^n$ be a standard chart on the real vector space V. If $[a_{ij}]$ is a constant positive definite matrix then the conditions

$$g(\partial/\partial x^i, \partial/\partial x^j) = a_{ij}, \qquad i, j=1, \cdots, n$$

determine a riemannian metric g on V. The corresponding riemannian connection is the standard connection. Consequently the radial vector field is

Received Feb. 15, 1973.