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RADIAL DISTRIBUTION OF ZEROS AND DEFICIENCY OF A CANONICAL PRODUCT OF FINITE GENUS

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1. Introduction. Edrei and Fuchs [1] proved the following

THEOREM A. Let f(z) be an entire function of finite order ρ , having only negative zeros. If $\rho > 1$, then $\delta(0, f) > 0$.

This reveals a quite interesting fact that a simple geometrical restriction is enough to make zero a deficient value. Edrei, Fuchs and Hellerstein [2] made the above result better. They gave a numerical bound

$$\delta(0, f) \ge \frac{A}{1+A}$$

with an absolute constant A>0. By a rough estimation their constant A satisfies A<0.0017. This is, of course, far from the best. There is still no reasonable conjecture for the best possible A.

They [2] gave the following result. (We state it here only in the case of genus one.)

THEOREM B. Let g(z) be a canonical product of genus one and having zeros $\{a_{\mu}\}$ in the sector

$$|\pi-\arg a_{\mu}| \leq \frac{\pi}{60}.$$

If the order of g is greater than one, then

$$\delta(0, g) \ge \frac{A}{1+A},$$

where A is the constant already mentioned.

Again $\pi/60$ is far from the best together with A. In this paper we shall prove the following

THEOREM 1. Let g(z) be a canonical product of genus q, having only negative

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