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ON THE GROWTH RATE OF COMPOSITIONS OF ENTIRE FUNCTIONS

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1. Let $f(z)$ be an entire function, $M(r, f)$ its maximum modulus on $|z|=r$ and $T(r, f)$ its Nevanlinna characteristic function. Recently Gross and Yang [4] proved the following:

Suppose that $f(z)$, $g(z)$ are entire functions such that

$$T(\alpha r, g)=o\{T(r, f)\} \quad as \quad r\to\infty \tag{1.1}$$

for some constant $\alpha>1$. Then for any non-constant entire function $h(z)$,

$$T(r, h\circ g)=o\{T(r, h\circ f)\} \quad as \quad r\to\infty$$

In this paper we shall consider the asymptotic behavior of the ratio $\log M(r, h\circ g)/\log M(r, h\circ f)$ replacing $T(r, \cdot)$ by $\log M(r, \cdot)$ in the above condition (1.1).

Our results are the following:

THEOREM 1. *Let $g(z)$ and $f(z)$ be entire functions such that*

$$\lim_{r\to\infty}\frac{\log M(\alpha r, g)}{\log M(r, f)}=0 \tag{1.2}$$

for some constant $\alpha>1$. Then for any non-constant entire function $h(z)$,

$$\lim_{r\to\infty}\frac{\log M(r, h\circ g)}{\log M(r, h\circ f)}=0.$$

THEOREM 2. *Let $g(z)$ and $f(z)$ be entire functions such that*

$$\lim_{r\to\infty}\frac{\log M(r, g)}{\log M(r, f)}=0. \tag{1.3}$$

Then for any non-constant entire function $h(z)$,

$$\varliminf_{r\to\infty}\frac{\log M(r, h\circ g)}{\log M(r, h\circ f)}=0.$$

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