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ON THE GROWTH OF ALGEBROID FUNCTIONS OF FINITE LOWER ORDER

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Dedicated to Professor Yukinari Tôki on his 60th birthday

1. In 1932 Paley [5] conjectured that
an entire function $g(z)$ of order λ satisfies

$$\varliminf_{r\to\infty}\frac{\log M(r, g)}{T(r, g)}\leqq\begin{cases}\dfrac{\pi\lambda}{\sin\pi\lambda} & \left(\lambda\leqq\dfrac{1}{2}\right),\\ \pi\lambda & \left(\lambda>\dfrac{1}{2}\right).\end{cases}$$

This conjecture was proved by Valiron [7] for $\lambda<1/2$. The first complete proof was given by Govorov [2]. A little later Petrenko [6] proved this conjecture for meromorphic functions of finite lower order. And D. F. Shea (cf. [1]) gave an improvement of Petrenko's theorem.

The purpose of this paper is to extend Shea's theorem to n-valued algebroid functions of finite lower order. Let $f(z)$ be an n-valued algebroid function, $f_j(z)$ the j-th determination of $f(z)$ and $T(r, f)$ the characteristic function of $f(z)$. We set

$$M(r, a, f)=\max_{|z|=r}\ \max_{1\leqq j\leqq n}\frac{1}{|f_j(z)-a|},\qquad a\neq\infty,$$

$$M(r, f)=M(r, \infty, f)=\max_{|z|=r}\ \max_{1\leqq j\leqq n}|f_j(z)|$$

and

$$\beta(a, f)=\varliminf_{r\to\infty}\frac{\log M(r, a, f)}{T(r, f)}.$$

We shall prove the following extension of Shea's theorem:

THEOREM 1. *Let $f(z)$ be an n-valued transcendental algebroid function of finite lower order μ and $\Delta(\infty)=\Delta$ the Valiron deficiency of $f(z)$ at ∞. Then we have*

$$\beta(\infty, f)\leqq n\pi\mu\{\Delta(2-\Delta)\}^{1/2}$$

if $\mu\geqq 1/2$ or $\mu<1/2$ and $\sin(\pi\mu/2)\geqq(\Delta/2)^{1/2}$, and

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