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INVARIANT SUBMANIFOLDS OF NORMAL CONTACT METRIC MANIFOLDS

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Introduction. Yano and Ishihara [8] have obtained conditions for an invariant submanifold of a normal contact metric manifold to be totally geodesic in the case of codimension 2.

In this connection, the purpose of the present note is to obtain some conditions for an invariant submanifold of codimension $p \geqq 2$ in a normal contact metric manifold to be totally geodesic. In §1, we shall recall notations and formulas for submanifolds and, in §2, definitions and some properties of a normal contact metric manifold. In §3, we shall give basic formulas for later use and obtain conditions for an invariant submanifold of a normal contact metric manifold to be totally geodesic under some additional conditions. In the last section, invariant submanifolds satisfying the condition $\tilde{R}(X, \xi)\cdot\alpha=0$ will be studied in a normal contact metric manifold. For tensor calculus we follow to notations employed in [1].

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§1. Submanifolds.

Let M be a manifold immersed in a Riemannian manifold $\bar{M}$. Because we shall describe only local properties, we may assume that M is small enough to be imbedded in $\bar{M}$ as a proper submanifold. Let $\mathfrak{X}(M)$ be the Lie algebra of vector fields on M and $\mathfrak{X}(M)^{\perp}$ the set of all vector fields perpendicular to M. We denote by $\bar{g}$ the metric tensor field on $\bar{M}$ and g the metric induced on M. $\bar{\nabla}$ denotes the covariant differentiation in $\bar{M}$ and ∇ the covariant differentiation in M determined by the induced metric g.

Let α be the second fundamental form of M. Then the formulas of Gauss and Weingarten are given by

$$(1.1) \qquad \bar{\nabla}_X Y=\nabla_X Y+\alpha(X, Y), \qquad X, Y\in\mathfrak{X}(M),$$

$$(1.2) \qquad \bar{\nabla}_X N=-A_N(X)+D_X N, \qquad X\in\mathfrak{X}(M), \qquad N\in\mathfrak{X}(M)^{\perp},$$

where $\bar{g}(A_N(X), Y)=\bar{g}(\alpha(X, Y), N)$ and $D_X N$ denotes the covariant derivative of

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