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QUATERNION KÄHLERIAN MANIFOLDS AND FIBRED RIEMANNIAN SPACES WITH SASAKIAN 3-STRUCTURE

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In a previous paper [6], we have studied fibred Riemannian spaces with Sasakian 3-structure and showed that there appears a kind of structure in the base space of a fibred Riemannian space with Sasakian 3-structure. In the present paper, we shall show that this kind of structure is what is called a quaternion Kählerian structure (See [1], [2], [3], [5], [7] and [9]).

In §1, we recall definitions and some properties of a fibred Riemannian space with Sasakian 3-structure for later use. In §2, we show that the base space of a fibred Riemannian space with Sasakian 3-structure admits a quaternion Kählerian structure defined in [5]. The last section is devoted to state some properties of a quaternion Kählerian manifold. Quaternion Kählerian manifolds will be studied a little bit in detail in [5].

Manifolds, mappings and geometric objects we consider are assumed to be differentiable and of class C^∞. The indices h, i, j, k run over the range $\{1, 2, \cdots, n\}$, the indices a, b, c, d, e over the range $\{1, 2, \cdots, n-3\}$ and the indices $\alpha, \beta, \gamma, \delta, \varepsilon$ over the range $\{1, 2, 3\}$. The summation convention will be used with respect to these three systems of indices.

§1. Fibred Riemannian spaces with Sasakian 3-structure.

In a Riemannian manifold $(\tilde{M}, \tilde{g})$ of dimension n with metric tensor $\tilde{g}$, let there be given a Killing vector ξ of unit length satisfying the condition

$$\tilde{\nabla}_j\tilde{\nabla}_i\xi^h = \xi_i\delta_j^h - \xi^h\tilde{g}_{ji}, \tag{1.1}$$

ξ^h being components of ξ and $\tilde{g}_{ji}$ components of $\tilde{g}$, where $\xi_i = \xi^h\tilde{g}_{hi}$ and $\tilde{\nabla}_j$ denote the Riemannian connection of $(\tilde{M}, \tilde{g})$. Then ξ is called a *Sasakian structure* or a *normal contact metric structure* in $(\tilde{M}, \tilde{g})$ (See [4] and [8]).

We now assume that $(\tilde{M}, \tilde{g})$ admits three Sasakian structures ξ, η and ζ which are mutually orthogonal and satisfy the conditions

$$[\eta, \zeta] = 2\xi, \qquad [\zeta, \xi] = 2\eta, \qquad [\xi, \eta] = 2\zeta.$$

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