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THE STRUCTURE OF BIVARIATE POISSON DISTRIBUTION

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0. Summary.

In this paper we consider the structure of two dimensional Poisson distribution. In section 1 the famous Poisson's theorem and an example are stated, in section 2 two dimensional Bernoulli distribution is defined and by the n independent convolution, two dimensional binomial distribution is defined as in one dimensional case and in section 3 the main result of this paper is stated that under some conditions the two dimensional binomial distribution approaches to two dimensional Poisson distribution and adding another condition it approaches to the distribution of independent type.

1. Poisson's theorem.

It is well known fact as Poisson's theorem that for given sequence of probabilities (p_n) such that $p_n \to 0$ $(n \to \infty)$ we have

$$P_n(m) - \frac{\lambda_n{}^m}{m!} e^{-\lambda_n} \to 0 \qquad \text{as} \quad n \to \infty$$

for all non-negative integer m where

$$\lambda_n = np_n, \qquad P_n(m) = \binom{n}{m} p_n{}^m (1-p_n)^{n-m}.$$

Furthermore if $np_n \to \lambda$ $(n \to \infty)$ then we have

$$P_n(m) \to \frac{\lambda^m}{m!} e^{-\lambda} \qquad (n \to \infty).$$

As an example of this theorem we consider a Bernoulli trial that event S occurs on a given unit space with probability p and S doesn't occur on this space with probability $1-p$. If we have n independent observations of the Bernoulli trial and we put the number of occurence of S in the n observations as X then the random variable X takes the value $0, 1, \cdots, n$ and the distribution is binomial:

$$P(X=k) = b(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k} \qquad (0 \leq k \leq n).$$

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