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PICARD CONSTANT OF A FINITELY SHEETED COVERING SURFACE

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§ 1. Introduction.

Let R be an open Riemann surface and $M(R)$ the set of non-constant meromorphic functions on R. Let f be a member of $M(R)$ and $P(f)$ the number of lacunary values of f. Let $P(R)$ be

$$\sup_{f\in M(R)} P(f).$$

This is called the Picard constant of R. It is known that $P(R)\geqq 2$ and $P(R)$ is conformally invariant. If R is an n-sheeted covering surface of $|z|<\infty$, then $2\leqq P(R)\leqq 2n$ [4].

In this paper we shall consider the following problem:

PROBLEM. Determine the Picard constant of a finitely sheeted covering surface of $|z|<\infty$.

This problem is very difficult to solve, in general. We shall restrict ourvelves to an n-sheeted covering surface R which is called regularly branched, that is, a surface which has no branch point other than those of order $n-1$.

Ozawa [5] has proved the following result:

If R is a two-sheeted covering surface of $|z|<\infty$ and if $P(R)=4$, then R is essentially equivalent to the surface defined by an algebroid function y such that $y^2=(e^H-\alpha)(e^H-\beta)$, where H is an entire function and α, β are constants satisfying $\alpha\beta(\alpha-\beta)\neq 0$.

Niino and Hiromi [1] have proved the following result:

If R is a three-sheeted regularly branched covering surface and if $P(R)\geqq 5$, then $P(R)=6$ and R is essentially equivalent to the surface defined by $y^3=(e^H-\alpha)\times(e^H-\beta)^2$, where H is an entire function and α, β and non-zero constants satisfying $\alpha\neq\beta$.

In § 2 we shall consider a preliminary result on $P(f)$.

In § 3 we shall prove a generalization of the above results.

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