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MINIMAL SURFACES IN A RIEMANNIAN MANIFOLD OF CONSTANT CURVATURE

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For surfaces in a 4-dimensional Riemannian manifold of constant curvature, the author [3] proved the following

THEOREM. *Let M be a 2-dimensional connected compact Riemannian manifold which is minimally immersed in a unit sphere of dimension 4. If the normal scalar curvature K_N is non-zero constant, then M may be regarded as a Veronese surface.*

In this paper, he generalizes the above theorem and proves the following

THEOREM. *Let M be a 2-dimensional connected compact Riemannian manifold which is minimally immersed in a $(2+\nu)$-dimensional unit sphere $S^{2+\nu}$. If the normal scalar curvature K_N is non-zero constant and the square of the second curvature k_2 is less then $K_N/4$, then M is a generalized Veronese surface.*

By a generalized Veronese surface we mean a surface defined by Ōtsuki [6].

§ 1. Preliminaries.

Let $\bar{M}$ be a $(2+\nu)$-dimensional Riemannian manifold of constant curvature $\bar{c}$ and M be a 2-dimensional Riemannian manifold immersed isometrically in $\bar{M}$ by the immersion x: $M \to \bar{M}$. $F(\bar{M})$ and $F(M)$ denote the orthonormal frame bundles over $\bar{M}$ and M respectively. Let B be the set of all elements $b=(p, e_1, e_2, e_3, \cdots, e_{2+\nu})$ such that $(p, e_1, e_2)\in F(M)$ and $(p, e_1, e_2, e_3, \cdots, e_{2+\nu})\in F(\bar{M})$ identifying $p\in M$ with $x(p)$ and e_i with $dx(e_i)$, $i=1, 2$. Then B is naturally considered as a smooth submanifold of $F(\bar{M})$. Let $\bar{\omega}_A$, $\bar{\omega}_{AB}=-\bar{\omega}_{BA}$, $A, B=1, 2, 3, \cdots, 2+\nu$, be the basic and connection forms of $\bar{M}$ on $F(\bar{M})$ which satisfy the structure equations:

$$(1.1)\qquad \begin{aligned} d\bar{\omega}_A &= \sum_B \bar{\omega}_{AB}\wedge\bar{\omega}_B, \\ d\bar{\omega}_{AB} &= \sum_C \bar{\omega}_{AC}\wedge\bar{\omega}_{CB}-\bar{c}\,\bar{\omega}_A\wedge\bar{\omega}_B. \end{aligned}$$

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