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CONSTANCY OF HOLOMORPHIC SECTIONAL CURVATURE IN ALMOST HERMITIAN MANIFOLDS

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Dedicated to Professor K. Yano on his 60th birthday

§ 1. Introduction.

Let (M, g, J) be an almost Hermitian manifold with almost complex structure tensor J and almost Hermitian metric tensor g. By R we denote the Riemannian curvature tensor; $R(X, Y)Z=\nabla_{[X,Y]}Z-[\nabla_X, \nabla_Y]Z$. The holomorphic sectional curvature $H(X)$ for a unit tangent vector X is the sectional curvature $K(X, JX)$ $=g(R(X, JX)X, JX)$. Let x be a point of M. If $H(X)$ is constant for every unit tangent vector X at x, (M, g, J) is said to be of constant holomorphic sectional curvature at x. If $H(X)$ is constant for every x and every tangent vector X at x, then (M, g, J) is said to be of constant holomorphic sectional curvature.

One of the main theorems is as follows:

THEOREM A. *Let* $\dim M=m=2n\geq 4$. *Assume that almost Hermitian manifold* (M, g, J) *satisfies*

$$g(R(JX, JY)JX, JZ)=g(R(X, Y)X, Z) \tag{1.1}$$

for every tangent vectors X, Y *and* Z. *Then,* (M, g, J) *is of constant holomorphic sectional curvature at* x, *if and only if*

$$R(X, JX)X \quad \text{is proportional to} \quad JX \tag{1.2}$$

for every tangent vector X *at* x.

The condition (1. 1) is satisfied in every Kählerian manifold or more generally in every K-space (=nearly Kählerian space, almost Tachibana space).

The condition (1. 2) itself has a geometric meaning. It is also stated as follows: Let σ be a holomorphic plane and let X, JX be in σ; then $R(X, JX)$ satisfies $R(X, JX)\sigma\subset\sigma$ and $R(X, JX)\sigma^{\perp}\subset\sigma^{\perp}$, where $\sigma^{\perp}$ denotes the orthocomplement of σ in the tangent space.

In § 2, as preliminaries we state some Propositions which give conditions for a Riemannian manifold to be of constant curvature.

In § 3, we prove Theorem A. Theorem A is concerned with point-wise constant

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