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ON ALMOST CONTACT AFFINE 3-STRUCTURES

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The almost quaternion structure has been studied by Ako [10], Bonan [1], Obata [6, 7] and one of the present authors [10]. The purpose of the present paper is to study almost contact affine 3-structures [2, 3, 4, 5, 8, 9] induced on hypersurfaces of an almost quaternion or quaternion manifold.

§ 1. Hypersurfaces of an almost quaternion manifold.

Let M^{4n} be an almost quaternion manifold, that is, a $4n$-dimensional differentiable manifold which admits a set of three tensor fields $\tilde{F}, \tilde{G}, \tilde{H}$ of type (1, 1) satisfying

$$\tilde{F}^2=-I, \qquad \tilde{G}^2=-I, \qquad \tilde{H}^2=-I,$$
$$\tilde{F}=\tilde{G}\tilde{H}=-\tilde{H}\tilde{G}, \qquad \tilde{G}=\tilde{H}\tilde{F}=-\tilde{F}\tilde{H}, \qquad \tilde{H}=\tilde{F}\tilde{G}=-\tilde{G}\tilde{F}, \tag{1. 1}$$

I denoting the identity tensor.

We first prove

LEMMA 1. 1. *There exists an almost Hermitian metric $\tilde{g}$ for the almost quaternion structure $\tilde{F}, \tilde{G}, \tilde{H}$, that is, a Riemannian metric $\tilde{g}$ satisfying*

$$\tilde{g}(\tilde{F}\tilde{X}, \tilde{F}\tilde{Y})=\tilde{g}(\tilde{X}, \tilde{Y}),$$
$$\tilde{g}(\tilde{G}\tilde{X}, \tilde{G}\tilde{Y})=\tilde{g}(\tilde{X}, \tilde{Y}), \tag{1. 2}$$
$$\tilde{g}(\tilde{H}\tilde{X}, \tilde{H}\tilde{Y})=\tilde{g}(\tilde{X}, \tilde{Y})$$

for arbitrary vector fields $\tilde{X}$ and $\tilde{Y}$ of M^{4n}.

Proof. Take an arbitrary Riemannian metric $\tilde{a}$ in M^{4n} and put

$$\tilde{b}(\tilde{X}, \tilde{Y})=\tilde{a}(\tilde{X}, \tilde{Y})+\tilde{a}(\tilde{F}\tilde{X}, \tilde{F}\tilde{Y}),$$

then we easily see that

$$\tilde{b}(\tilde{F}\tilde{X}, \tilde{F}\tilde{Y})=\tilde{b}(\tilde{X}, \tilde{Y})$$

since $\tilde{F}^2=-I$. We next put

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