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DIFFERENTIABLE SOLUTIONS OF ALGEBRAIC EQUATIONS ON MANIFOLDS

BY SAMUEL I. GOLDBERG AND NICHOLAS C. PETRIDIS

1. Introduction. Yano [7] introduced the notion of an *f-structure*, which is a non-null (1, 1) tensor field f of constant rank r on a C^∞ manifold of dimension $r+m$, satisfying $f^3+f=0$. An almost complex and an almost contact structure are particular cases of an f-structure the existence of an f-structure being equivalent to a reduction of the structural group of the tangent bundle to $U(r/2)\times O(m)$. They were studied by various authors ([1], [2], [6], etc.) with particular focus on the case of globally framed structures [2]. Extending the concept of an f-structure, Goldberg and Yano [3] introduced the notion of a polynomial structure on a manifold.

An f-structure is a particular case of an almost product structure [7], [8]. The purpose of this paper is to point out the close relation of the polynomial structures on manifolds and the almost product structures as defined by Walker [8]. In § 2 it is shown that any polynomial structure generates an almost product structure. From this follow necessary and sufficient conditions for a distribution to be globally framed and for a manifold to be parallelizable. In § 3 reductions of the structural group of the tangent bundle of a polynomial structure are obtained, similar to that for f-structures (see [7]). It is shown that for any polynomial structure with structure polynomial decomposable into distinct irreducible quadratic factors over the reals R that there is an underlying almost complex structure. In § 5 an analogue of the normal f-structures [2] is examined which is more general in the sense that the tensor field f is not required to satisfy an algebraic equation.

2. Almost product structure. Let M be a differentiable manifold. A C^∞ tensor field f of type (1, 1) on M is said to define a *polynomial structure* if f satisfies the algebraic equation

$$P(x)=x^m+a_m x^{m-1}+\cdots+a_2x+a_1I=0, \tag{2.1}$$

where I is the identity mapping and $f^{m-1}(p), f^{m-2}(p), \cdots, f(p), I$ are linearly independent for every $p\in M$. Clearly, f is non-singular if and only if $a_1\neq 0$. The polynomial $P(x)$ is called the *structure polynomial.* If $P(x)=x^2+I$ we have an almost complex structure.

An *almost product structure* on a differentiable manifold M is a system of differentiable distributions $T_1, T_2, \cdots, T_k$ such that

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