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THE RUDIN KERNEL AND THE EXTREMAL FUNCTIONS IN HARDY CLASSES

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1. In [8], we have been concerned with the Rudin kernel and some associated reproducing kernels on compact bordered Riemann surfaces. The Rudin kernel which is analytic on $\bar{S}$ (i.e. the closure of S) is characterized by the following reproducing property on a compact bordered Riemann surface S: Let $H_2(S)$ be the class of analytic functions f on S such that $|f|^2$ has harmonic majorants on S. For fixed $x, t \in S$,

$$f(x)=\frac{1}{2\pi}\int_{\partial S} f(\tau)\overline{R_t(\tau, x)}\frac{\partial g(\tau, t)}{\partial \nu}ds_\tau \qquad \text{for all} \quad f\in H_2(S).$$

Here ∂S is the relative boundary of S, $g(\tau, t)$ is the Green function of S with pole at t and the derivative is taken along the inner normal. Furthermore the adjoint L-kernel $\mathcal{L}_t(\tau, t)$ is characterized by the following relation:

$$\overline{R_t(\tau, x)}\frac{\partial g(\tau, t)}{\partial \nu}ds_\tau=\frac{1}{i}\mathcal{L}_t(\tau, x) \qquad \text{along} \quad \partial S. \tag{1.1}$$

Here $\mathcal{L}_t(\tau, x)$ is an analytic differential on $\bar{S}$ except for a simple pole at x with residue 1. However as we have pointed out (cf. [8], Lemma 3.1), the Rudin kernel on a compact bordered Riemann surface does not characterize completely the Rudin kernel. In the present paper, we shall be concerned with some further properties of the Rudin kernel on an arbitrary Riemann surface and a general region in the plane. Let S be an arbitrary open Riemann surface. For fixed $x, t(\in S)$, let $\{S_n\}_{n=0}^{\infty}$ $(S_0 \ni x, t)$ be a regular exhaustion of S. Let $R_t^{(n)}(\tau, x)$ denote the Rudin kernel of S_n with respect to t and x. Let $g_n(\tau, t)$ denote the Green function of S_n with pole at t. Let $H_p(S)$ $(p>0)$ be the class of analytic functions f on S for which $|f|^p$ has harmonic majorants on S. For any $f\in H_p(S)$, let u_f denote the least harmonic majorant of $|f|^p$. Then we define H_p-norm on S with respect to t by $\|f\|_{S,p}^t=u_f(t)^{1/p}$ for any fixed point t on S.

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2. At first, the following theorem is fundamental:

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