A. INATOMI
KŌDAI MATH. SEM. REP
25 (1973), 34—36

ON GALOIS THEORY OF CENTRAL SEPARABLE ALGEBRAS OVER ARTINIAN RINGS

BY AKIRA INATOMI

Let A be a separable algebra over the center C of A and B a subring of A. Let G be a finite group of automorphisms of A and A/B an *outer G-Galois extension* in the sense of Miyashita [6]. In [4], we had the following result: If C is a separable algebra over the center R of B, then C is a G^*-*Galois extension* of R and $G \cong G^*$, where G^* is the group of automorphisms of C induced by G.

In this note, we shall show the following result: If C is an *artinian* ring, then C is a G^*-*Galois extension* of R and $G \cong G^*$.

Let A' be a ring such that the center of A' is C and A' is projective as a C-module. Let T be a subring of A. Since A is a central separable algebra, A is projective as a C-module ([1], Th. 2.1). Hence we may regard T as a subring of $A \otimes_C A'$ by the natural ring monomorphism.

LEMMA 1. *If* $V_A(T)$[1] $=C$, *then* $V_{A \otimes_C A'}(T)=A'$.

Proof. Since A' is projective as a C-module, there exists a C-free module F such that A' is inbedded in F by a C-monomorphism $f: A' \to F$. We have the exact sequence

$$0 \longrightarrow A \otimes_C A' \xrightarrow{f^*} A \otimes_C F,$$

where $f^* = 1 \otimes f$. We can regard $A \otimes_C A'$ as a two-sided A-module and $A \otimes_C F$, too. Then f^* is a two-sided A-module monomorphism. Since A is a separable algebra over C, C is a direct summand of A as a C-module ([1], Th. 2.1). Hence we have $A = C \oplus D$, where D is a C-submodule of A. Then,

$$A \otimes_C A' = A' \oplus D \otimes_C A', \qquad A \otimes_C F = F \oplus D \otimes_C F$$

and $f^{*-1}(F) = A'$. We take any element z of $V_{A \otimes_C A'}(T)$ and we set $x = f^*(z)$. Let $[y_i]_{i \in I}$ be a base of F, then $x = \sum_i x_i \otimes y_i$, where $x_i \in A$ and $x_i = 0$ for almost all i. Since $tz - zt = 0$ for all $t \in T$ and f^* is a two-sided A-module monomorphism, we have

Received December 10, 1971.

1) We denote by $V_A(T)$ the commutor of T in A.