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FIBRED SPACES WITH ALMOST COMPLEX STRUCTURES

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Dedicated to Professor Yosio Mutō on his sixtieth birthday

Introduction.

Many papers on the theory of submersion, together with immersions, have been published in recent years (e.g. [1], [4], [8], [9], [17]). A mapping σ from a manifold $\tilde{M}^n$ onto a manifold M^m is called a *submersion* if its differential σ_* is of rank m at any point of $\tilde{M}^n$, where n is larger than m. It seems, generally speaking, that there are two directions of investigating submersions. One is to discuss the existence of a submersion in a given manifold and the other is to study a manifold in which a submersion is assumed to be given a priori. The submersion has also been studied as a fibred space. The concept of a fibred space has been used, since 1922, in unified field theories and in the theory of projective connections.

The purpose of the present paper is to study fibred spaces with a projectable Riemannian metric and a projectable almost complex structure. In §§1 and 2 definitions and lemmas are stated in the most general case for the later use. We discuss in §3, by use of tensor analysis, the properties of a fibred Riemannian manifold in detail. The structure equations for a fibred space are prepared in §4. In §5, we assume that $\tilde{M}$ and fibres are both of even dimensional and we introduce in $\tilde{M}$ an almost complex structure. First we assume that each fibre is an invariant subspace of $\tilde{M}$ and next we treat with more general case. For the case in which the dimension of a fibre is odd, especially 1-dimensional, see [7], where an almost contact structure is introduced in $\tilde{M}$.

§1. Preliminaries.

Let $\tilde{M}$ and M be differentiable[1] manifolds of dimension n and m respectively, where n is larger than m. We assume that there is given a differentiable submersion σ from $\tilde{M}$ to M, that is, σ is a differentiable mapping from $\tilde{M}$ onto M whose differential σ_* is of rank m at each point $\tilde{P}$ of $\tilde{M}$. Therefore, the complete inverse image $\mathcal{F}_P$ of $P \in M$ is an $n-m$ dimensional closed submanifold of $\tilde{M}$. We call $\mathcal{F}_P$ a *fibre* over P. Throughout this paper we assume that every fibre is

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1) Differentiability is always assumed to be of C^∞.