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# ON HYPERSURFACES WITH CONSTANT SCALAR CURVATURE IN A RIEMANNIAN MANIFOLD OF CONSTANT CURVATURE

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## Introduction.

It is well known that Liebmann [7] and Süss [15] proved that a compact convex hypersurface with constant mean curvature in a Euclidean space is a sphere. The tool used in the proof of this theorem is an integral formula containing the first mean curvature of the hypersurface. The position vector and the support function play an important role in the proof. In his paper [5], Hsiung has established an integral formula expressing the relation between the $j$-th mean curvature and the $(j+1)$-st mean curvature of a compact hypersurface in a Euclidean space, which is a generalization of that of Süss. By applying the integral formula, Hsiung proved a theorem which gives a sufficient condition for a compact hypersurface in a Euclidean space to be a sphere. This generalizes the theorem of Liebmann and Süss. The study in this line has been carried out by Amur [2], Reilly [10], Shahin [13], Yano and Tani [17] and others.

On the other hand, Simons [14] has recently done an important and suggestive contribution to the study of minimal submanifolds in a Riemannian manifold, in which he has given a formula for the Laplacian of the square of the norm of the second fundamental form of the submanifold. Under the stimulus of the Simons' study, Carmo, do Chern and Kobayashi [3], and Nomizu and Smyth [9], using the similar formula to that of Simons, have obtained some theorems on a compact minimal submanifold or a complete hypersurface with constant mean curvature in a Riemannian manifold of constant curvature.

The purpose of this paper is to generalize, by applying a formula of Simons' type to a compact hypersurface with constant scalar curvature in a Riemannain manifold of constant curvature, the theorem of Liebmann and Süss from the different point of view. We prove that a compact hypersurface of non-negative curvature and with constant scalar curvature in a Euclidean space is a sphere.

In § 1, we state preliminaries and in § 2 we obtain the main theorem (Theorem 2. 3) stating a compact hypersurface $M$ with constant scalar curvature in a simply connected space form $M^{n+1}(c)$ is totally umbilic or has exactly two distinct and constant principal curvatures under suitable conditions. Making use of this pro-

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