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INVARIANT SUBMANIFOLDS OF AN *f*-MANIFOLD WITH COMPLEMENTED FRAMES

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Introduction. Recently, invariant hypersurfaces of a Kaehler manifold with constant holomorphic sectional curvature and invariant Einstein (or η-Einstein) submanifolds of normal contact or cosymplectic manifolds with constant ϕ-sectional curvature have been studied by several authors [2], [3], [4], [7]. Blair [1] has quite recently defined and studied $\mathcal{S}$-manifolds and $\mathcal{T}$-manifolds which reduce, in special cases, to normal contact manifolds and cosymplectic manifolds respectively.

Generalizing the notion of η-Einstein contact manifolds, we shall define, in §1, η-Einstein $\mathcal{S}$-manifolds and $\mathcal{T}$-manifolds and obtain some formulas giving curvature tensors for $\mathcal{S}$-manifolds and $\mathcal{T}$-manifolds with constant f-sectional curvature. In §2, we shall define f-invariant and invariant submanifolds in an $\mathcal{S}$-manifold or a $\mathcal{T}$-manifold and study invariant η-Einstein submanifolds of codimension 2 in an $\mathcal{S}$-manifold or a $\mathcal{T}$-manifold of constant f-sectional curvature. In the last section, we shall study f-invariant hypersurfaces in a certain $\mathcal{S}$-manifold or a $\mathcal{T}$-manifold. The authors wish to express their deep gratitude to Professor S. Hokari for his kind guidances and encouragement.

1. *f*-manifolds with complemented frames.

Let $\tilde{M}=\tilde{M}^{2n+s}$ be a manifold with an $\tilde{f}$-structure of rank $2n$. In the sequel, we assume that $n>1$. If there exist in $\tilde{M}$ vector fields $\underset{x}{\tilde{\xi}}$ $(x=1, \cdots, s)$ such that

$$
\begin{aligned}
&\underset{x}{\tilde{\eta}}(\underset{y}{\tilde{\xi}})=\delta_{xy},\\
(1.1)\qquad &\tilde{f}\underset{x}{\tilde{\xi}}=0, \qquad \underset{x}{\tilde{\eta}}\circ\tilde{f}=0,\\
&\tilde{f}^2=-1+\sum_x \underset{x}{\tilde{\xi}}\otimes\underset{x}{\tilde{\eta}},
\end{aligned}
$$

where $\underset{x}{\tilde{\eta}}$ are duals to $\underset{x}{\tilde{\xi}}$, then the $\tilde{f}$-structure is said to be with complemented frames $\underset{1}{\tilde{\xi}}, \cdots, \underset{s}{\tilde{\xi}}$ or simply to be with complemented frames. If $\tilde{M}$ has an $\tilde{f}$-structure with complemented frames, then there exists in $\tilde{M}$ a Riemannian metric $\tilde{G}$ such that

$$
(1.2)\qquad \tilde{G}(\tilde{X}, \tilde{Y})=\tilde{G}(\tilde{f}\tilde{X}, \tilde{f}\tilde{Y})+\tilde{\Phi}(\tilde{Y}, \tilde{Y}),
$$

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