NICKEL, P. A.
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THE LINEAR OPERATOR METHOD AND LINEAR $\mathfrak{S}$ TOPOLOGIES

BY PAUL A. NICKEL

In 1955, Sario [7] published his basic paper on the Linear Operator Method. The objective there was the construction of harmonic functions p with the behavior of a prescribed singularity function s near the ideal boundary. This objective was described in terms of constructing a harmonic function p for which $p-s$ has a regular, or normal behavior near the boundary; that is, $p-s$ is itself the image of an operator L which is reminiscent of a Dirichlet operator and is called normal [1], [6], and [7]. In this sense, the harmonic function p is thought of as an extension of the singularity function s, modulo a regular singularity function defined on a regular boundary neighborhood W' of the Riemann surface W. It is shown in [7] that except for constants, this extension is unique, and furthermore, that if the difference of two singularity functions is regular, then each will have the same harmonic extension, except for a constant.

In [5], Rodin and Sario have placed this extension problem in the natural setting of quotient spaces, wherein they have given an elegant solution phrased in terms of establishing that the natural mapping $p \to s = p|_{W'}$ will induce an algebraic isomorphism. The purpose of the present effort is to seek linear topologies under which this basic natural mapping will in fact induce a topological isomorphism.

1. Notation. We consider a regular boundary neighborhood W' of an open Riemann surface W. The boundary of W' is denoted by α and the closure of W', a union of bordered Riemann surface with border α, is denoted $\bar{W}'$. We call the linear space ot all harmonic functions on W by $H(W)$, and with a slight abuse of consistency we define $H(W')$ to be the linear space consisting of all functions which are harmonic on W', continuous on $\bar{W}'$, and have vanishing flux on the ideal boundary β of W.

When $C(\alpha)$ denotes the linear space of continuous functions on α, we let L: $C(\alpha) \to H(W')$ be a normal operator in the usual sense ([1], [5], [6], or [7]). The main existence theorem of Sario [7] phrased in algebraic language, is

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