KŌDAI MATH. SEM. REP.
24 (1972), 259–269

FUNCTIONAL CENTRAL LIMIT THEOREMS FOR STRICTLY STATIONARY PROCESSES SATISFYINC THE STRONG MIXING CONDITION

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1. Summary.

The object of this paper is to prove the functional central limit theorems for strictly stationary processes satisfying the strong mixing condition under the same assumptions in Ibragimov [3]. The results generalize those of Davydov [2].

2. Main results.

Let $\{\xi_n;\ n=0, \pm 1, \pm 2, \cdots\}$ be a strictly stationary process with $E\xi_j=0$, satisfying the strong mixing (s. m.) condition, i.e.,

$$\sup_{A\in\mathfrak{M}_{-\infty}^{a},\, B\in\mathfrak{M}_{a+s}^{\infty}} |P(AB)-P(A)P(B)|=\alpha(s)\to 0 \qquad (s\to\infty), \tag{1}$$

where $\mathfrak{M}_a^b$ denotes the σ-algebra generated by $\{\xi_j;\ j=a, \cdots, b\}$. Write $S_n=\xi_1+\cdots+\xi_n$ and $\sigma^2=E\xi_0^2+2\sum_{j=1}^{\infty}E\xi_0\xi_j$. Let $D=D[0,1]$ be the space of functions x on $[0,1]$ that are right-continuous and have left-hand limits, and let $\mathcal{D}$ be the σ-field of Borel sets for the Skorokhod topology (cf. [1]). When $0<\sigma<\infty$, we define random elements $X_n(t)$ of D by

$$X_n(t, \omega)=\frac{1}{\sigma\sqrt{n}}S_{[nt]}(\omega), \qquad 0\leqq t\leqq 1;\ n=1, 2, \cdots \tag{2}$$

The following theorems imply that functional central limit theorems hold under the same conditions of theorems 1.6 and 1.7 in [3] which assure the validity of central limit theorems.

THEOREM 1. *If ξ_j's are bounded, i.e., $|\xi_j|<C<\infty$ with probability one and if*

$$\sum_{n=1}^{\infty}\alpha(n)<\infty \qquad \textit{and} \qquad \alpha(n)\leqq\frac{M}{n\log n}, \tag{3}$$

then $\sigma^2<\infty$. If $\sigma>0$ and if X_n is defined by (2), then the distribution of X_n converges weakly to Wiener measure W on $(D, \mathcal{D})$.

Received May 20, 1971.