KŌDAI MATH. SEM. REP.
24 (1972), 195—202

AN ARCLENGTH PROBLEM FOR m-FOLD SYMMETRIC UNIVALENT FUNCTIONS

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1. Introduction. Let S denote the class of functions

$$f(z)=z+\sum_{n=2}^{\infty}a_nz^n$$

which are analytic and univalent in the unit disk Δ: $|z|<1$. Let S^* denote the subclass of S for which $f(z)$ is starlike, that is

$$\mathrm{Re}\left[\frac{zf'(z)}{f(z)}\right]>0 \qquad (z\in\Delta).$$

Let C denote the subclass of S for which $f(z)$ is convex, that is

$$\mathrm{Re}\left[\frac{zf''(z)}{f'(z)}+1\right]>0 \quad (z\in\Delta).$$

Let K denote the subclass of S for which $f(z)$ is close-to-convex, that is

$$\mathrm{Re}\left[\frac{zf'(z)}{h(z)}\right]>0 \qquad (z\in\Delta).$$

where $h(z)$ is starlike. These classes are related by the proper inclusions $C\subset S^*\subset K\subset S$.

A function $f(z)$ analytic in Δ is said to be m-fold symmetric ($m=1, 2, \cdots$) if

$$f(e^{2\pi i/m}z)=e^{2\pi i/m}f(z).$$

In particular, every $f(z)$ is 1-fold symmetric and every odd $f(z)$ is 2-fold symmetric. Let S_m denote the subclass of S for which $f(z)$ is m-fold symmetric. A simple argument shows that $f\in S_m$ is characterized by having a power series of the form

$$f(z)=z+a_{m+1}z^{m+1}+a_{2m+1}z^{2m+1}+\cdots.$$

We similarly define S_m^*, C_m and K_m.

For $f\in S$ and $0<r<1$, let

$$L_r(f)=\int_{|z|=r}|f'(z)|\,|dz|$$

Received September 10, 1971.