KŌDAI MATH. SEM. REP.
24 (1972), 146—167

ON CANONICAL STRATIFICATIONS

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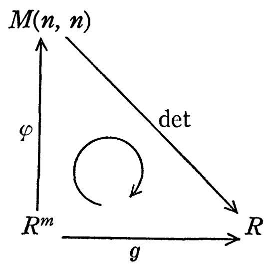
§ 0. Introduction.

It is well-known that every compact manifold can be imbedded into a Euclidean m-space R^m for some m. Furthermore Nash [7] proved that for a closed connected smooth manifold M, smoothly imbedded in R^m, there is a polynomial map f: $R^m \rightarrow R^q$ for some q such that M is a connected component of $f^{-1}(0)$. A polynomial map f is an ordered set $(g_1, g_2, \cdots, g_q)$ of polynomial functions.

On the other hand, by a simple calculation, we have the following

PROPOSITION A. *Every polynomial can be expressed in a form of determinant of a certain square matrix whose entries are monomials of degree* 1 *or* 0. *More precisely, for any polynomial function* g: $R^m \rightarrow R$, *there is a positive integer* n *and an affine imbedding* φ *of* R^m *into the space* $M(n, n)$ *of all* $n \times n$ *real matrices such that the following diagram is commutative*:



(This was communicated to the author by T. Ishikawa).

REMARK. For the given polynomial map $f=(g_1, \cdots, g_q)$: $R^m \rightarrow R^q$, we take the positive integer n common to all g_i.

On account of the above facts every closed connected smooth manifold can be imbedded into $M(n, n)$ for some n and is expressed as the intersection of the q affine m-spaces $\varphi_i(R^m)$ and $\det^{-1}(0)$ in $M(n, n)$. Thus it is meaningful to study the set of zeros of det: $M(n, n) \rightarrow R$, which is the same as the set of singular matrices, or the set of matrices with rank $r<n$. More generally we consider, in this paper, the set of $n \times m$ real matrices, $n \leqq m$, with rank $r<n$.

Received March 11, 1971.