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MINIMALITY IN FAMILIES OF SOLUTIONS OF $\Delta u=Pu$ ON RIEMANN SURFACES

By Kwang-nan Chow

I. Introduction.

Consider a Riemann surface R and the space $HD(R)$ of all harmonic functions with finite Dirichlet integral. The monotone closure of $HD(R)$ is denoted by $\widetilde{HD}(R)$. C. Constantinescu and A. Cornea in 1958 started the study of minimal functions in $\widetilde{HD}(R)$. In 1960, Nakai ([2], also cf. [7]) introduced a representing measure on the Royden boundary Γ associated to R and a kernel on $R\times\Gamma$ which serve to represent $\widetilde{HD}(R)$. One significant result is that $\widetilde{HD}$-minimal functions correspond to atoms on Γ.

It was Ozawa [5] who first considered the solutions of $\Delta u=Pu$ on R where P is a nonnegative density. Glasner and Katz [1] have recently shown that solutions of $\Delta u=Pu$ on R can also be studied in terms of their behavior on Γ. Using their machinery, one can obtain analogues of Nakai's results for the space $PE(R)$ of solutions with finite energy integral and its monotone closure $\widetilde{PE}(R)$. In particular, a representing measure on Γ and a kernel on $R\times\Gamma$ can be constructed for solutions so that $\widetilde{PE}$-minimal functions can be characterized analogously.

In view of this, a natural question is: what is the relation between the $\widetilde{HD}$- and $\widetilde{PE}$-minimality? Or equivalently, if a point on Γ is atomic with respect to one measure, will it be atomic with respect to another? In this paper it is shown that the answer is virtually yes. This answer is encouraging because it suggests that there is a topological property of Γ which can be associated with $\widetilde{HD}$- or $\widetilde{PE}$-minimality. Finding such a property would have important implications in the study of quasi-conformal or quasi-isometric invariants.

As a remark, all the results in this paper can be carried over to Riemannian manifolds.

II. Preliminaries.

II 1. We consider an open Riemann surface R and the equation $\Delta u=Pu$ on R, where P is a nonnegative density. For simplicity, solutions of $\Delta u=Pu$ will be called solutions. Let $M(R)$ be the Royden algebra associated with R which is the

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