ON DEFICIENCIES OF AN ENTIRE ALGEBROID FUNCTION

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§1. Niino and Ozawa [1, 2] proved some interesting results for entire algebroid functions. A typical one is the following:

Let f(z) be a two-valued entire transcendental algebroid function and a_1, a_2 and a_3 be different finite numbers satisfying

$$\sum_{j=1}^{3} \delta(a_j, f) > 2.$$

Then at least one of $\{a_j\}$ is a Picard exceptional value of f.

They also proved in the three- and four-valued cases that a more weaker condition on deficiencies, under a "non-proportionality" condition, implies the existence of Picard exceptional values (Theorem 1 in [2]).

In this paper we shall discuss the five-valued case and establish the similar conclusions as in Theorem 1 in [2] under a different assumption on deficiencies (see also Ozawa [3]). Those are the following:

THEOREM 1. Let f(z) be a five-valued transcendental entire algebroid function defined by an irreducible equation

$$F(z, f) \equiv f^{5} + A_{4}f^{4} + A_{3}f^{3} + A_{2}f^{2} + A_{1}f + A_{0} = 0,$$

where A_4 , A_3 , A_2 , A_1 and A_0 are entire functions. Let a_j , $j=1, \dots, 6$, be different finite numbers satisfying

$$\sum_{j=1}^{6} \delta(a_j, f) + \delta(a_m, f) + \delta(a_n, f) > 7$$

for every pair $m, n \ (m \neq n), m, n = 1, \dots, 6$, where $\delta(a_j, f)$ indicates the Nevanlinna-Selberg deficiency of f at a_j . Further assume that any four of $\{F(z, a_j)\}$ are not linearly dependent. Then one of $\{a_j\}_{j=1}^6$ is a Picard exceptional value of f.

THEOREM 2. Let f(z) be the same as in Theorem 1. Let $\{a_j\}_{j=1}^{\gamma}$ be different finite numbers satisfying

$$\sum_{j=1}^{6} \delta(a_j, f) + \delta(a_m, f) + \delta(a_n, f) > 7$$

for every pair $m, n \ (m \neq n), m, n = 1, \dots, 6, and$

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