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PSEUDO-UMBILICAL SURFACES IN EUCLIDEAN SPACES

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Recently, the author introduced the notion of αth curvatures of first and second kinds for surfaces in higher dimensional euclidean space [2, 3]. The main purpose of this paper is to study these curvatures more detail. In §1, we derive some integral formulas for the αth curvatures of first and second kinds. In §2, we get some applications of these formulas to pseudo-umbilical surfaces.

§1. Integral formulas for αth curvatures.

Let M^2 be an oriented closed Riemannian surface with an isometric immersion x: $M^2 \to E^{2+N}$. Let $F(M^2)$ and $F(E^{2+N})$ be the bundles of orthonormal frames of M^2 and E^{2+N} respectively. Let B be the set of elements $b=(p, e_1, e_2, \cdots, e_{2+N})$ such that $(p, e_1, e_2) \in F(M^2)$ and $(x(p), e_1, \cdots, e_{2+N}) \in F(E^{2+N})$ whose orientation is coherent with that of E^{2+N}, identifying e_i with $dx(e_i)$, $i=1, 2$. Then $B \to M^2$ may be considered as a principal bundle with fibre $O(2) \times SO(N)$ and $\tilde{x}$: $B \to F(E^{2+N})$ is naturally defined by $\tilde{x}(b)=(x(p), e_1, \cdots, e_{2+N})$.

The structure equations of E^{2+N} are given by

$$(1)\qquad \begin{aligned} &dx=\sum_A \omega'_A e_A, \qquad de_A=\sum_B \omega'_{AB} e_B, \\ &d\omega'_B=\sum_B \omega'_B \wedge \omega'_{BA}, \qquad d\omega'_{AB}=\sum_C \omega'_{AC} \wedge \omega'_{CB}, \qquad \omega'_{AB}+\omega'_{BA}=0, \\ &\qquad A, B, C, \cdots=1, 2, \cdots, 2+N, \end{aligned}$$

where ω'_A, ω'_{AB} are differential 1-forms on $F(E^{2+N})$. Let ω_A, ω_{AB} be the induced 1-forms on B from ω'_A, ω'_{AB} by the mapping $\tilde{x}$. Then we have

$$(2)\qquad \omega_r=0, \qquad \omega_{ir}=\sum_j A_{rij}\omega_j, \qquad A_{rij}=A_{rji}, \qquad i, j, \cdots=1, 2; \qquad r, t, \cdots=3, \cdots, 2+N.$$

From (1), we get

$$(3)\qquad d\omega_i=\sum_j \omega_j \wedge \omega_{ji}, \qquad d\omega_{AB}=\sum_C \omega_{AC} \wedge \omega_{CB},$$

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