KŌDAI MATH. SEM. REP.
23 (1971), 343—350

ON THE CONCURRENT VECTOR FIELDS OF IMMERSED MANIFOLDS

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Let R^m be an m-dimensional Riemannian manifold[1] with covariant derivative D and let $x: M^n \to R^m$ be an immersion of an n-dimensional manifold M^n into R^m. A vector field X in R^m over M^n is called a *concurrent vector field*[2] if we have $dx+DX=0$, where dx denotes the differential of the immersion x. In particular, if X is a normal vector field of M^n in R^m, then the vector field X is called a *concurrent normal vector field.*

The main purpose of this paper is to study the behavior of the concurrent vector fields of immersed manifolds and also find a characterization of the concurrent vector fields with constant length.

§ 1. Preliminaries.

Let R^m be an m-dimensional Riemannian manifold with covariant derivative D. By a frame $e_1, \cdots, e_m$, we mean an ordered set of m orthonormal vectors $e_1, \cdots, e_m$ in the tangent space at a point of R^m. The frame $e_1, \cdots, e_m$ defines uniquely a dual coframe $\bar{\omega}_1, \cdots, \bar{\omega}_m$ in the cotangent space and vice versa. The fundamental theorem of local Riemannian geometry says that in a neighborhood U of a point p there exists uniquely a set of linear differential forms $\bar{\omega}_{AB}$ satisfying the conditions:

$$\bar{\omega}_{AB}+\bar{\omega}_{BA}=0, \tag{1}$$

and

$$d\bar{\omega}_A=\sum \bar{\omega}_B \wedge \bar{\omega}_{BA}, \tag{2}$$

where here and in the sequel the indices $A, B, \cdots$ run over the range $\{1, \cdots, m\}$. The linear differential forms $\bar{\omega}_{AB}$ are called the connection forms and the covariant derivative DX of a vector field $X=\sum X_A e_A$, is given by

Received October 31, 1970.

1) Manifolds, mappings, tensor fields and other geometric objects are assumed to be differentiable and of class C^∞.

2) In [3], a vector field X is called a concurrent vector field if there exists a function f such that $dx+D(fX)=0$, but in this paper, we adopt the above definition.