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THE LAW OF THE ITERATED LOGARITHM FOR STATIONARY PROCESSES SATISFYING MIXING CONDITIONS

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0. Summary.

The law of the iterated logarithm for various stochastic sequences has long been studied by many authors. Recently, Iosifescu proved in [5] that the law holds for stationary sequences satisfying the uniformly strong mixing condition and Reznik showed in [8] that the one is also valid for stationary processes satisfying the strong mixing condition. But, the conditions used in [5] and [8] are slightly stringent. The purpose of this paper is to weaken those conditions, that is, to prove the law under as similar as possible requirements to the conditions in [3].

1. Definitions and notations.

Let $\{x_j, -\infty<j<\infty\}$ be processes which are strictly stationary and satisfy one of the following conditions:

$$\text{(I)} \qquad \sup_{A\in\mathcal{M}_{-\infty}^{k}, B\in\mathcal{M}_{k+n}^{\infty}} \frac{1}{P(A)}|P(A\cap B)-P(A)P(B)|=\varphi(n)\to 0 \quad (n\to\infty)$$

or

$$\text{(II)} \qquad \sup_{A\in\mathcal{M}_{-\infty}^{k}, B\in\mathcal{M}_{k+n}^{\infty}} |P(A\cap B)-P(A)P(B)|=\alpha(n)\to 0 \quad (n\to\infty),$$

where $\mathcal{M}_a^b$ denotes the σ-algebra generated by events of the type

$$\{(x_{i_1}, \cdots, x_{i_k})\in E\}, \qquad a\leqq i_1<\cdots<i_k\leqq b$$

and E is a k-dimensional Borel set. In line with [4], we shall call Condition (I) the uniformly strong mixing (u.s.m.) condition and (II) the strong mixing (s.m.) codition.

In what follows, we assume that all processes $\{x_j\}$ are strictly stationary, $Ex_j=0$ and $Ex_j^2<\infty$. We shall agree to denote by the letter K_i a quantity bounded in absolute value.

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