INFINITE TENSOR PRODUCTS OF VON NEUMANN ALGEBRAS, I

By Yoshiomi Nakagami

§ 1. Introduction.

For the purpose of studying lattice systems of quantum statistical mechanics and representations of CCR and CAR, infinite tensor products of von Neumann algebras due to von Neumann [12] have been frequently used as shown in [1], [4], [6] and others. The problems of types of infinite tensor products of von Neumann algebras have been investigated by many authors [2], [3], [7], [9], [11]. Infinite tensor products of normal positive linear functionals have been studied by Takeda [10] and symmetric states of infinite tensor products have been recently studied by Størmer [8]. Most of these results have been treated in the cases of incomplete infinite tensor products and of factors.

When we study infinite tensor products of von Neumann algebras, we set a problem what kind of relations has a finite normal trace given in the infinite tensor product of von Neumann algebras, with a finite measure on an infinite product space of some topological spaces corresponding to given von Neumann algebras? We encounter this problem in the course of studying infinite dimensional measures such as weak distributions, cylindrical measures and integrations of functionals. In the present paper we prepare some results on infinite tensor products of operators and those of normal positive linear functionals, which are defined in complete infinite tensor products of Hilbert spaces, in order to give some informations on that problem. By utilizing the results of this paper a partial answer will be given in the subsequent paper*⁾ of the same title. In Theorem 3.1 some conditions by which infinite tensor products of operators can be defined will be discussed, and in Theorem 3.2 the conditions that infinite tensor products of operators belong to a given infinite tensor product of von Neumann algebras or to its commutor will be obtained. In Theorem 4.1 a sufficient condition that infinite tensor products of normal positive linear functionals can be defined will be given by introducing a concept of characteristic numbers. The similar results together with the necessary condition for finite normal traces will be given in Theorem 4.2 with the aid of coupling operators. Beside this theorem will indicate a finite part of infinite tensor product of von Neumann algebras as shown in Corollary 4.2.