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ON A SYSTEM OF LINEAR ORDINARY DIFFERENTIAL EQUATIONS RELATED TO A TURNING POINT PROBLEM

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§ 1. Introduction.

1° In order to analyse the so called turning point problem, sometimes the given equation will be reduced to a simpler type. If the given equation, however, has a "complicated" turning point, it will be investigated in several domains separately, where the original equation behaves in a quite different manner, and each solution obtained in the corresponding domain will be *matched* with the solutions in adjacent domains by adequate methods. Iwano [2] analysed how to divide the domain where the equation is defined and how to reduce the equation in each of the divided domains. For instance, the equation with a turning point at the origin

$$\varepsilon\frac{dy}{dx}=\begin{bmatrix}0 & 1\\ x^3-\varepsilon & 0\end{bmatrix}y$$

can be changed by a transformation $y=\mathrm{diag}[1, x^{3/2}]u$ to

$$(x^{-3}\varepsilon)x^{3/2}\frac{du}{dx}=\left\{\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}+(x^{-3}\varepsilon)\begin{bmatrix}0 & 0\\ -1 & -\dfrac{3}{2}x^{1/2}\end{bmatrix}\right\}u$$

in a domain $M_1|\varepsilon|^{1/3}\leqq|x|\leqq\delta_0$; by transformations $x=\varepsilon^{1/3}\xi$, $y=\mathrm{diag}[1, \varepsilon^{1/2}]v$ to

$$\varepsilon^{1/6}\frac{dv}{d\xi}=\begin{bmatrix}0 & 1\\ \xi^3-1 & 0\end{bmatrix}v$$

in a domain $M_2|\varepsilon|^{1/2}\leqq|x|\leqq M_1|\varepsilon|^{1/3}$; and by transformations $x=\varepsilon^{1/2}\eta$, $y=\mathrm{diag}[1, \varepsilon^{1/2}]w$ to

$$\frac{dw}{d\eta}=\left\{\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}+\varepsilon^{1/2}\begin{bmatrix}0 & 0\\ \eta^3 & 0\end{bmatrix}\right\}w$$

in a domain $|x|\leqq M_2|\varepsilon|^{1/2}$. Here δ_0 is a small constant and M_i $(i=1, 2)$ are large

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