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MARKOV CHAINS WITH RANDOM TRANSITION MATRICES

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Introduction.

Let P^t ($t=1, 2, 3, \cdots$) be the transition matrix from epoch $t-1$ to t of a Markov chain with a finite state space S, and $\alpha^{(n)}$ ($n=0, 1, 2, \cdots$) be the probability distribution at n. Then we have

$$(*)\qquad \alpha^{(n)}=\alpha^{(0)}P^1\cdots P^n.$$

Now we assume that $\alpha^{(0)}$ and P^t are mutually independent random variables and that $\alpha^{(n)}$ is defined by (*). Then $\{\alpha^{(n)}\}$ is a Markov process on the space of probability distributions on S. ($\alpha^{(n)}$ represents the probability distribution at n, starting with the initial distribution $\alpha^{(0)}$ and following to the *random* transition matrices P^t.) Such a process will be called a "*Markov chain with random transition matrices*" (*M.C. with R.T.M.*).

The author intended to generalize ordinary Markov chains as briefly mentioned above, by the following reason.

Markov chains have been applied in many fields, and one of their applications is in the analysis or prediction of market shares. Many authors have worked with so-called Markov brand-switching models, in which $\alpha^{(n)}$ represents the market shares at epoch n and P^t represents the transition matrix from epoch $t-1$ to t. In many cases they assume that these Markov chains (they consider that $\alpha^{(n)}$ is the distribution of a Markov chain at step n) are stationary. However some other authors have given warnings of the failure of stationarity and of other defects of these models (see e.g. A. S. C. Ehrenberg [1]). The author thinks that one of the causes of the warnings is in the assumption that $\alpha^{(0)}$ and P^t are *a priori* given (known) and hence have no stochastic fluctuation. The transition matrix P^t reflects the choices of purchasers, and so it is essentially stochastic. Hence it seems to be natural to consider that P^t and $\alpha^{(n)}$ are random variables. The most simple stochastic model for market shares is the model using our M.C. with R.T.M.

In this paper, properties of M.C. with R.T.M., in particular moments of $\alpha^{(n)}$ and conditions for the convergence (in law) of $\alpha^{(n)}$, are given. And we classify stationary and irreducible M.C. with R.T.M. into three groups; ergodic chains, aperiodic and non-ergodic chains, and periodic chains. Finally we prove some ergodic theorems.

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