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THE STRONG CONVERSE THEOREM IN THE DECODING SCHEME OF LIST SIZE L

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1. Introduction.

The coding theorem, discovered by Shannon [5], states that information can be transmitted with arbitrarily small error probability by means of words lengthening. A question will arise on what will become of an asymptotic behavior of an error probability. Denoting by M the number of input messages and by N the length of corresponding input words, the rate is defined as $R=(1/N)\log M$. If we lengthen the word with a fixed rate, the error probability approaches exponentially zero. For a broad class of channels its first exponential error-bound was given by Fano (1956). Its precise upper estimate was obtained by Gallager [4], and its precise lower estimate was obtained by Shannon, Gallager, and Berlekamp [6].

On the other hand, the weak converse theorem for Shannon's coding theorem was proved by Feinstein [3] using Fano's inequality. It states that if the rate is above the channel capacity, for a sufficiently large N the error probability is positive. Wolfowitz [7] proved the strong converse theorem; there exists a positive constant K such that there does not exist $M=e^{NC+K\sqrt{N}}$ input messages such that its error probability is below $\lambda>0$. In this paper we derive the strong converse theorem in the decoding scheme of list size L. The rate is defined as

$$R=\frac{1}{N}\log\frac{M}{L}.$$

If $R>C+\varepsilon$, then the average error probability approaches one. The decoding scheme of list size L which was mentioned in Shannon, Gallager, and Berlekamp [6], is that the decoder, rather than mapping the output words into a single message, maps it into a list of messages. If the transmitted source message is not on the list of decoded message, we say that a list decoding error has occurred.

2. Channel and list decoding.

Let input alphabets be $i=1,\cdots,I$ ($I\leqq a$) and output alphabets be $j=1,\cdots,J$ ($J\leqq a$). Let X_N be the set of all input words of length N that can be transmitted, and let Y_N be the set of all output words of length N that can be received. Let $P(y\mid x_m)$, for $x_m\in X_N$ and $y\in Y_N$, be the conditional probability of received word y, given that

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